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Exploring Students' Mathematical Reasoning Through Discovery Learning with Gamification Media on the Pythagorean Theorem

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ABSTRACT

Reasoning is an important foundation in solving mathematical problems. Reasoning is a thinking process that allows students to understand concepts, connect ideas, and draw logical conclusions. This study aimed to explore students' mathematical reasoning in Discovery Learning supported by gamification media on the Pythagorean Theorem. The study used a qualitative case study approach involving two eighth-grade students at a junior high school in Malang. Data were obtained through observation, interviews, and mathematical reasoning tests, then analyzed using the Miles and Huberman model. The findings revealed that Discovery Learning, supported by gamification media, created an interactive learning experience that enabled students to gradually explore concepts in context. High-ability students demonstrated strong reasoning skills, including analyzing problems, constructing arguments, applying inductive and deductive reasoning, making generalizations, and drawing conclusions. Low-ability students also showed improvement in several reasoning indicators, particularly in analyzing problems, applying deductive reasoning, and developing mathematical arguments during the learning process.

Keywords: reasoning; discovery learning; gamification; pythagorean theorem

INTRODUCTION

Mathematical reasoning is one of the key skills in mathematics education because it involves the process of logical thinking in understanding, connecting, and drawing conclusions from mathematical concepts (Hačatrjana & Namsone, 2024; Harini et al., 2025). Reasoning is demonstrated not only through the final answers students obtain but also through the thought processes they employ in solving problems (Rohmah et al., 2020). Lithner (Lithner, 2008) explains that reasoning is both a process and the outcome of that process, as evidenced by the strategies students use to solve mathematical problems.

National Council of Teachers of Mathematics (NCTM) (National Council of Teacher of Mathematics (NCTM), 2000) states that mathematical reasoning encompasses the ability to make conjectures, construct arguments, evaluate solutions, and draw logical conclusions. Mathematical reasoning is not only related to the ability to obtain answers but also involves the ability to analyze problem situations, develop mathematical arguments, use inductive and deductive reasoning, make generalizations, and draw well-reasoned conclusions (Kasmer & Kim, 2011). Therefore, mathematical reasoning can be understood as the process through which students build mathematical relationships. Thus, exploring students' reasoning processes is crucial for understanding how students construct their mathematical understanding.

However, several study findings indicate that students' mathematical reasoning in Indonesia tends to be at a relatively low level (Negara et al., 2022; Sari et al., 2024). Students still struggle to understand the relationships between mathematical concepts. Ramdan & Roesdiana (2022) state that students' low level of mathematical reasoning is evident in their inability to make generalizations, perform mathematical manipulations, use patterns and relationships to analyze problems, and draw accurate conclusions. Furthermore, students tend to solve problems procedurally, without being able to explain the mathematical reasoning behind their steps (Andrianti & Hidayah, 2025). This situation suggests that mathematics learning may not yet fully provide sufficient opportunities for the development of students' reasoning.

This issue is also evident in the material on the Pythagorean Theorem. Many students are able to apply the Pythagorean Theorem formula to solve problems, but are not yet able to explain the relationship between the sides of a right triangle that underlies the formula (Khoerunnisa & Sari, 2021; Mulyani et al., 2025). Students tend to memorize the solution procedure without understanding the mathematical relationships underlying the concept-formation process. In fact, the Pythagorean Theorem allows students to develop reasoning through activities such as relating side lengths, making conjectures, verifying relationships, and drawing conclusions. Therefore, the Pythagorean Theorem is a relevant topic for exploring students' mathematical reasoning during the learning process.

One learning model that provides students with the opportunity to demonstrate their reasoning process is Discovery Learning. The Discovery Learning model places students at the center of the learning process by having them gradually discover concepts (Chusni et al., 2021). In the learning process, students are encouraged to observe, gather information, process data, verify, and draw conclusions (Pongpalilu, 2023). These stages are believed to provide opportunities for reasoning activities such as analyzing situations, formulating hypotheses, evaluating relationships, and drawing conclusions based on available information (Masfingatin & Murtafiah, 2020). Thus, Discovery Learning can serve as a learning context that supports students' exploration of mathematical reasoning.

To support student engagement in the learning process, Discovery Learning can be combined with gamification. Gamification is the application of game elements in learning activities to increase student participation and engagement (Lestari et al., 2024; Luo, 2022; Riar et al., 2022; Septian et al., 2025). Elements such as challenges, feedback, and rewards can encourage students to more actively explore and solve mathematical problems (Alt, 2023). These conditions can also render students' reasoning processes more observable during learning. Therefore, gamification is not only expected to increase learning motivation but also supports the emergence of students' mathematical reasoning activities during concept discovery.

Research on students' reasoning abilities regarding the Pythagorean theorem has been extensively conducted, such as by Rifai & Prihatnani (2020), Ramdani & Prayitno (2023), and Shofiyyah & Qohar (2022). These studies generally emphasize the correctness of final answers or the level of reasoning achieved, but provide limited insight into how students' reasoning processes emerge during learning. On the other hand, studies on the use of the Discovery Learning model and gamification media have also been conducted and have shown positive impacts on student learning outcomes, such as problem-solving skills

(Darmawan & Suparman, 2019) and conceptual understanding (Maharani et al., 2026; Saharani & Abadi, 2024). However, these studies mostly focus on learning outcomes rather than the development of students' reasoning processes across learning stages.

This suggests a gap in understanding how students construct and develop mathematical reasoning during gamification-assisted Discovery Learning, particularly in the context of the Pythagorean Theorem. Based on this gap, the research question addressed in this study is "How does students' mathematical reasoning develop during gamification-assisted Discovery Learning on the Pythagorean Theorem?" This study aims to examine students' mathematical reasoning during gamification-assisted Discovery Learning on the Pythagorean Theorem, focusing on both students' final solutions and the reasoning processes that emerge at each stage of learning. The findings are expected to contribute to a deeper understanding of students' mathematical reasoning and to support the design of learning activities that foster more active mathematical thinking.

RESEARCH METHOD

This study employs a qualitative approach using a case study design to explore students' mathematical reasoning in Discovery Learning supported by gamification media on the Pythagorean Theorem. A qualitative approach was chosen because the study aims to provide an in-depth description of students' mathematical reasoning processes. A case study was selected to gain a more detailed understanding of students' mathematical reasoning within a specific learning context.

The learning took place over three sessions using the Discovery Learning model, which includes the stages of stimulation, problem statement, data collection, data processing, verification, and generalization. During the learning process, each student used gamification media via a smartphone. The gamification media were developed using the Unity Game Engine as an adventure game that incorporates activities for discovering and applying the concepts of the Pythagorean Theorem. The media is equipped with gamification elements such as points, badges, challenges, and feedback to encourage student engagement throughout the learning process. Throughout the learning process, students are encouraged to explore mathematical relationships, formulate hypotheses, verify them, and draw conclusions based on the activities.

The research subjects were 2 students selected through purposive sampling based on their results in mathematical reasoning tests, student activities during learning, and interview results. Based on this data, two students with different mathematical reasoning abilities were selected: one with high reasoning ability and one with low reasoning ability. The selection of subjects was conducted to gain a deeper understanding of students' mathematical reasoning characteristics during the learning process. Thus, an in-depth analysis was conducted on the reasoning processes of both subjects while solving problems on the Pythagorean Theorem material.

The research instruments included observation sheets, a mathematical reasoning test, and interview guidelines. The observation sheets were used to observe the implementation of instruction and student activities during the learning process. The mathematical reasoning test (Figure 1) consists of open-ended questions on the Pythagorean Theorem designed to reveal students' reasoning. The interview guide is used to delve deeper into students' thought processes based on their answers to the mathematical reasoning test.

1. A weather radar is operating at an observation station. From this position, the radar detects two weather balloons: Balloon X and Balloon Y. Balloon X is directly above the radar at an altitude of 21 meters. Meanwhile, Balloon Y is at the same altitude as Balloon X but is 35 meters away from the radar. Determine the distance between Balloon X and Balloon Y!
2. A carpenter is building a wall shelf with a triangular frame. He has three pieces of wood measuring 12 dm, 13 dm, and 5 dm in length. Check whether these three pieces of wood can be arranged to form a right triangle. Show your calculations!

Figure 1. Test Questions

Table 1. Mathematical Reasoning Indicators

Reasoning Indicator	Description
Analyzing and evaluating problem situations	Students can identify relevant information and relationships between parts to determine solution steps.
Developing, evaluating, and supporting or refuting mathematical arguments	Students can construct mathematical arguments and evaluate them to determine whether they can be accepted or rejected with proper justification.
Using inductive and deductive reasoning to establish or support mathematical relationships	Students can discover patterns through inductive reasoning to form conjectures, then use deductive reasoning to prove or strengthen the mathematical relationships obtained.
Formulating, evaluating, and making generalizations	Students can formulate general statements based on patterns or regularities observed in several cases and evaluate the validity of the generalizations made.
Drawing and supporting conclusions in various contexts	Students can formulate conclusions based on the problem-solving process they have carried out and provide logical reasons to support those conclusions.

Data analysis was conducted qualitatively using the Miles and Huberman (1994) model, which includes data reduction, data display, and conclusion drawing. Analysis of student responses was based on the mathematical reasoning indicators by Kasmer & Kim (Kasmer & Kim, 2011) as shown in Table 1, and presented in a descriptive narrative to illustrate the characteristics of reasoning for each indicator. Data validity was ensured through triangulation, which involved comparing data from tests, observations, and interviews to ensure the consistency of the research findings.

RESULTS AND DISCUSSION

The learning process was conducted using the Discovery Learning model, supported by adventure-based gamification media, comprising four levels of activities, ranging from exploring the concept of triangles to applying the Pythagorean Theorem to solve contextual problems. The use of gamification media enabled students not only to focus on calculation procedures but also to engage in exploring geometric relationships through interactive activities. During the learning process, students are guided to observe, formulate hypotheses, verify, and draw conclusions based on the learning experiences gained through the media. These conditions allow for the emergence of various forms of mathematical reasoning among students throughout the learning process.

In the stimulation stage, students first explore the game-based media to understand the concept of a right triangle. Through gamification, students are guided to identify the hypotenuse (the sloping side) in a right triangle. After completing the media exploration, students are presented with a problem involving a right triangle in different base positions (Figure 2). This stage reveals variations in students' initial reasoning when determining the triangle's hypotenuse. Some students identified the hypotenuse based on its position opposite the right angle, while others relied on visual cues, selecting the side that appeared longest. These differing responses indicate that the stimuli within the gamified media encouraged students to begin analyzing the relationships among the triangle's elements more critically.

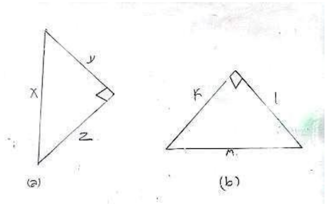

Figure 2. Representations of Right Triangles with Different Base Positions

During data collection and processing stages, it was observed that students examined a diagram of a right triangle with a square attached to each of its sides. From this illustration, students began to establish relationships between the sides based on the areas of the squares formed on each side. At this stage, there were several square sizes, so students were guided to identify a pattern: the area of the square on the hypotenuse equals the sum of the areas of the two squares on the legs. The pattern discovered then serves as the basis for students to form an initial hypothesis regarding the mathematical relationship in right triangles. This finding indicates that students not only engage in visual observation but also begin to develop an understanding of the quantitative relationship between sides through the analysis of area patterns. Thus, the emerging process leads to the development of inductive reasoning in identifying mathematical patterns from various observed cases.

In the verification stage, students engaged in proving the Pythagorean Theorem through manipulative activities within a gamified media. Students arranged several congruent right triangles to form two equal squares. From this activity, students observed that although the figures' arrangement differed, the total area formed remained the same. Students then relate this finding to the Pythagorean Theorem, namely the relationship that the square of the hypotenuse equals the sum of the squares of the legs. In addition to this proof activity, students also solve contextual problems available on the platform to test their conceptual understanding. In this process, students begin to connect the information provided with concepts they have previously acquired. In the generalization stage, students formulated conclusions based on the entire sequence of activities they had completed. Students stated that in right triangles, the relationship in which the square of the hypotenuse equals the sum of the squares of the legs holds true, as a generalization of the results from their exploration in the gamified media.

Based on observations during the learning process, differences in students' mathematical reasoning characteristics were evident in their understanding and problem-solving of the Pythagorean Theorem. Some students demonstrated the ability to logically connect concepts and verify the mathematical relationships obtained, while others still struggled to analyze information and construct appropriate mathematical models. These differences in characteristics were then analyzed in greater depth through mathematical reasoning tests and interviews with two research subjects: S1, a student with high reasoning ability, and S2, a student with low reasoning ability. The analysis aimed to describe students' mathematical reasoning processes during gamification-assisted Discovery Learning on the Pythagorean Theorem.

Mathematical Reasoning of High-Achieving Students

In gamification-assisted Discovery Learning, Student S1 demonstrated active engagement in identifying the information provided. S1 did not merely focus on the triangle's visual appearance but also connected the information to the appropriate geometric concepts. This was evident when the student was asked to analyze representations of right triangles with different base positions (Figure 2). In that activity, S1 stated, "*Even though the triangle is rotated, the hypotenuse remains the side opposite the right angle, not the one that appears slanted*." This response indicates that S1 had moved beyond visual reasoning toward conceptual reasoning based on geometric definitions. Furthermore, the ability to establish mathematical relationships logically also began to emerge when S1 stated, "*For triangle (a), since the side is opposite the right angle, then it is the hypotenuse, so it applies*." S1's statement reflects the use of deductive reasoning in determining the relationships between sides based on the concept of a right triangle.

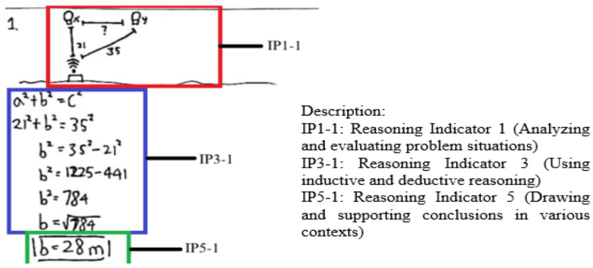

Figure 3. Analysis of S1's Mathematical Reasoning Indicators on Question 1

Consistent with the findings from the learning activities, Student S1 also demonstrated the ability to analyze and evaluate the given problem. In Question 1 (Figure 3), S1 was able to identify relevant information and modeled it as a right-angled triangle, as shown in Figure 3 (section IP1-1). This modeling process shows that S1 first constructed a representation before applying formulas, rather than directly using procedures. This is supported by the interview results in Transcript 1.

Transcript 1

Researcher	:	*"After reading the question, what did you understand?"*
S1	:	*"I understood the information by forming a right-angled triangle. The side length a represents the distance between the radar and balloon X, which is 21 m; b represents the distance between balloons X and Y; and c represents the distance from balloon Y to the airborne radar, which is 35 m."*

Additionally, the ability to use inductive and deductive reasoning to establish mathematical relationships also begins to emerge in Figure 3 (IP3-1). This finding is reinforced by the interview results in Transcript 2.

Transcript 2

Researcher	:	*"How did you know that the Pythagorean Theorem could be applied to this problem?"*
S1	:	*"Because after I drew it, I saw a right angle, so I could relate the sides using* $a^2 + b^2 = c^2$ *to verify the result."*

The statement in Transcript 2 indicates that S1 first identified the triangular shape of the given problem, then selected the Pythagorean Theorem to solve it. S1 also drew conclusions from the calculated results, as shown in Figure 3 (IP5-1). Transcript 3 shows that S1 not only obtained the calculated results but also interpreted them within the context of the given problem.

Transcript 3

Researcher : *"What conclusion did you reach, and how did you verify the final result of your calculations?"*

S1 : *"The distance between Balloon X and Balloon Y is 28 m. To verify the answer, I checked that all my calculations were correct and there were no errors"*

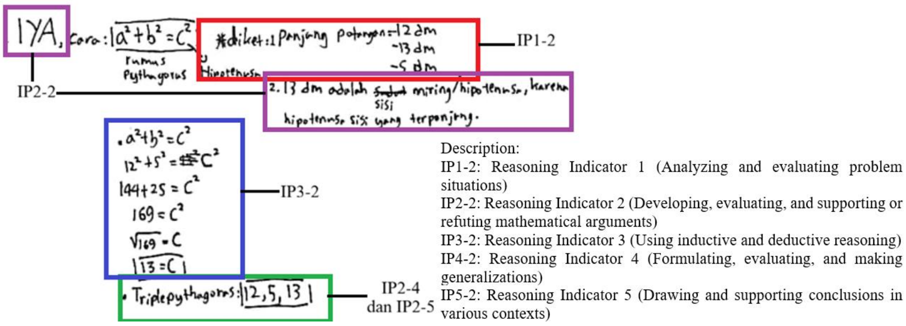

Figure 4. Analysis of S1's Mathematical Reasoning Indicators on Question 2

In Question 2 (Figure 4), S1 demonstrated the ability to hypothesize that the side lengths of $12\ dm, 5\ dm$, and $13\ dm$ could form a right triangle with 13 dm as the hypotenuse because it is the longest side (Figure 4 IP2-3). This hypothesis was then clarified through the interview results in Transcript 4.

Transcript 4

Researcher : *"Why did you choose 13 as the hypotenuse?"*

S1 : *"Because in a right triangle, the longest side is opposite the right angle and is the longest side, so since 13 is the longest, the hypotenuse is 13."*

The statement in Transcript 4 indicates that S1 used the triangle's side lengths as the basis for an initial argument about its shape. To prove this hypothesis, S1 then employed deductive reasoning (Figure 4 IP3-2) by applying the Pythagorean Theorem to verify the relationships between the sides. This is further supported by the interview results in Transcript 5.

Transcript 5

Researcher : *"What was your purpose in using the Pythagorean Theorem?"*

S1 : *"The Pythagorean Theorem is used to ensure that the lengths of those sides can form a right triangle."*

In the final stage, S1 was able to draw conclusions and generalize the results into the concept of Pythagorean triples (Figure 4, IP4 and IP5). This is reinforced by the interview presented in Transcript 6.

Transcript 6

Researcher : *"What conclusion did you reach?"*

S1 : *"Since the sum of the squares of the legs equals the square of the hypotenuse, it satisfies the Pythagorean Theorem, so 12, 5, and 13 form a right triangle."*

Researcher : *"Why did you write down* $12, 5, 13$ *as a Pythagorean triple?"*

S1 : *"Because 12, 5, and 13 are three natural numbers that satisfy the Pythagorean theorem, so they fit the definition of a Pythagorean triple."*

The results of the interview in Transcript 6 indicate that S1 not only summarized the calculation results but also identified a general pattern in the relationship among the three numbers that satisfy the Pythagorean Theorem. This reflects the ability to formulate generalizations, namely linking specific results to a more general concept, namely Pythagorean triples.

Based on these findings, S1's mathematical reasoning demonstrates a coherent sequence, ranging from problem modeling and forming conjectures to verifying mathematical relationships using the Pythagorean Theorem and drawing conclusions and generalizing to the concept of Pythagorean triples. This indicates that S1 can consistently connect the problem-solving process to more general mathematical concepts. These research findings reinforce previous findings that students with high reasoning abilities can evaluate their thinking processes and construct generalizations from identified patterns (Triwardani et al., 2018). Furthermore, S1's ability to link contextual information to mathematical representations aligns with the findings of Rosyidah et al. (2021), who emphasize that good reasoning is demonstrated through accuracy in connecting information with relevant concepts. In the context of solving geometry problems, this ability also supports the view of Moh & Dian (2018) that students with high reasoning skills can formulate solution steps systematically, logically, and in accordance with concepts.

Mathematical Reasoning Among Students with Low Proficiency

In gamification-assisted Discovery Learning, S2 demonstrated engagement in learning activities, but the mathematical reasoning displayed was still inconsistent. In the early stages, S2 tended to use a visual approach to understand the elements of a triangle without relating them to the concept of a right angle. This was evident when S2 provided a justification for Figure 2: "*In Figure 2 (a), y is the hypotenuse as seen in the figure*." This statement indicates that S2 still relied on visual perception and had not yet referred to the mathematical properties of geometric objects. As the learning process progressed, S2 began to develop an understanding of the relationships between the sides of a triangle. This is evident when S2 stated, "*In figure (b), the relationship* $k^2 + l^2 = m^2$ *applies*." This statement indicates that S2 is beginning to connect visual information with mathematical concepts, although it remains procedural and does not yet provide a rationale for the statement.

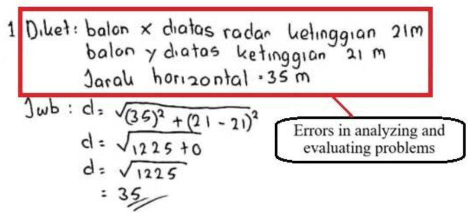

Figure 5. Analysis of S2's Mathematical Reasoning Indicators on Question 1

The findings from S2's mathematical reasoning test on Question 1 (Figure 5) show that S2 was unable to identify information accurately. Consequently, the mathematical model constructed did not align with the problem situation, leading to errors in the problem-solving process. The interview results in Transcript 7 reinforce these findings.

Transcript 7

Researcher	:	*"What do you understand from the given problem?"*
S2	:	*"I understand that the distance of balloons X and Y from the radar is 21 m and the distance between balloons X and Y is 35 m. But here I'm confused, because my calculation also gives a result of 35 m, so it doesn't form a right triangle."*

The statement in Transcript 7 indicates that S2 misinterpreted the information. The confusion that arose when the calculation result matched one of the known values also suggests that S2 did not properly evaluate the model. Thus, the difficulty experienced by S2 lies in the analysis and modeling stages, not in the calculation procedure. This indicates that the indicators of analyzing and evaluating problem situations, as well as evaluating mathematical arguments, have not been optimally met.

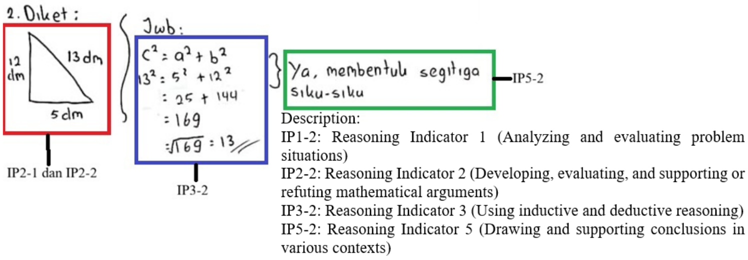

Figure 6. Analysis of S2's Mathematical Reasoning Indicators on Question 2

In Question 2 (Figure 6), S2 began to demonstrate the ability to identify geometric elements and form hypotheses (Figure 6 IP1-2 and IP1-2). S2 determined that the longest side is the hypotenuse based on visual observation of the side lengths.

Transcript 8

Peneliti	:	*"How did you determine and place the lengths of each side in that triangle?"*
S2	:	"Since 13 is the longest length, it becomes the hypotenuse, so c = 13. Then, for 5 and 12, those are the legs because they are shorter than 13; I just adjusted them visually, 12 is longer and 5 is shorter."

S2's statement in Transcript 8 shows that S2 determined the hypotenuse based on the longest side, which is 13 dm, and then placed the 12 dm and 5 dm sides as the legs with the help of a visual representation. This indicates that S2 used visual representations to understand the relationships among the triangle's sides, which served as the foundation for constructing an initial mathematical model. To prove this hypothesis, S2 used the Pythagorean theorem (Figure 7 IP3-2). This indicates that S2 has used inductive and deductive reasoning to examine the argument constructed. In addition, S2 concluded that the three sides form a right triangle, indicating achievement of the indicator for drawing conclusions. However, S2 has not yet demonstrated the ability to generalize more general concepts.

Based on these findings, the mathematical reasoning of S2 exhibits characteristics that are inconsistent and still developing, particularly in the early stages of analyzing and modeling problems. S2 tends to use a visual approach to understanding situations, which can lead to difficulties linking information to relevant concepts. This results in errors when constructing mathematical models, even though, procedurally, S2 is actually capable of performing calculations correctly. However, the findings also indicate the development of reasoning, particularly inductive and deductive reasoning. S2 students are beginning to formulate hypotheses based on observed patterns and verify them using the Pythagorean Theorem. These findings align with the research by Aizah et al (2025) and Musriroh et al. (2021), which states that students with low reasoning ability tend to struggle with understanding information and interpreting models of the given problems. Furthermore, Disnawati et al. (2023) and Maziidah et al. (2025) also demonstrated that students with low reasoning skills can apply formulas correctly but have not yet interpreted the results obtained within the context of the problem.

Based on the analysis conducted, a comparison of mathematical reasoning between S1 and S2 is presented in Table 2. The findings indicate that variation in students' reasoning primarily reflects the consistency and depth of their reasoning processes. Students with higher mathematical ability tend to demonstrate more structured and stable reasoning patterns, whereas students with lower ability show reasoning that is still developing and not yet consistent across problem-solving stages. This suggests that the quality of mathematical reasoning is influenced not only by computational ability, but also by the capacity to connect information, construct arguments, and evaluate solutions critically (Lithner, 2008; Syarifuddin et al., 2019)

Table 2. Differences in Mathematical Reasoning Between S1 and S2

Mathematical Reasoning Indicators	S1 (High Proficiency)	S2 (Low Proficiency)
Analyzing and evaluating problem situations	Able to consistently identify information and relationships between parts	Able to identify information, but not consistently and with some misinterpretations
Developing and evaluating mathematical arguments	Able to construct and evaluate arguments logically	Able to construct arguments, but does not yet conduct a thorough evaluation
Using inductive and deductive reasoning	Able to use inductive and deductive reasoning in an integrated manner	Begins to demonstrate inductive reasoning and is able to use deductive reasoning, but is not yet consistent
Formulating and making generalizations	Able to formulate generalizations based on patterns	Not yet able to formulate generalizations clearly
Drawing conclusions in the context of a problem	Able to draw conclusions appropriate to the context	Conclusions are still limited to mathematical results and are not yet fully contextual

In the Discovery Learning process supported by gamification, differences in reasoning development are evident in how students process and transform information. Students with higher ability tend to construct and use multiple representations, both visual and symbolic, as a basis for deductive justification. This reflects reflective reasoning, where mathematical concepts are understood as interconnected structures that can be applied across different contexts (Tisngati & Genarsih, 2021). Such reasoning patterns indicate flexible conceptual connections and higher-order thinking skills, including the ability to construct and evaluate mathematical arguments (Altindis, 2025; Lithner, 2008; Moh & Dian, 2018)

Meanwhile, students with lower ability tend to rely on visualizing objects when analyzing information. When faced with problems without explicit visualizations, students tend to create inaccurate representations of geometric shapes. This indicates that students' primary difficulty lies not in computational processes but in their ability to conceptualize mathematical relationships. These findings support the view of Dahlan et al. (2025), who argue that misconceptions about the Pythagorean Theorem often arise when students rely on visual perception rather than conceptual properties. Nevertheless, partial development of reasoning is still observed, particularly in the use of inductive and deductive approaches in later tasks, although these remain inconsistent and not yet well integrated (Star & Stylianides, 2013).

In this study, exploratory learning experiences served as a space that facilitated the emergence of variations in students' mathematical reasoning. The Discovery Learning model provides students with a space to develop their mathematical reasoning skills at each stage. The stimulation and problem statement stages facilitate the development of the ability to analyze information and make initial conjectures (Noer et al., 2020). Furthermore, during the data collection and data processing stages, students begin to organize information, allowing mathematical patterns to form implicitly (Noer et al., 2020; Yerimadesi et al., 2023). The verification stage serves as a validation space, where students test their conjectures using the formal principles of the Pythagorean Theorem as a form of deductive reasoning (Noer et al., 2020). The generalization stage requires students to formulate general relationships from the results of their exploration (Noer et al., 2020; Yerimadesi et al., 2023).

In the learning process, gamification media helps students understand the Pythagorean Theorem through interactive visualizations and immediate feedback (Dehghanzadeh et al., 2024). This is evident from research findings on students with low ability, who, despite difficulties in analyzing information in the early stages, were still able to reach the generalization stage after receiving assistance in the form of feedback and visualizations presented in the media. This indicates that gamification media not only support procedural understanding but also facilitate the gradual development of students' mathematical reasoning. This can be explained by the views of Kozma (1991) and Juhaevah (2024), who suggest that the media serve not merely as an aid but as an instrument that shapes students' thinking through repeated interaction. The integration of Discovery Learning and gamification is indicated to support the development of students' mathematical reasoning by facilitating progression from problem exploration to generalization, while interactive feedback and visual scaffolding help maintain engagement and strengthen conceptual understanding across different levels of ability. Nevertheless, this study is limited by its focus on a single mathematical topic and its qualitative design, which may limit the generalizability of the findings.

CONCLUSION

Based on the research findings, it can be concluded that learning using the Discovery Learning model, supported by gamification media, supports the development of students' mathematical reasoning regarding the Pythagorean Theorem, with distinct characteristics observed across subjects. High-ability students tend to meet all indicators of mathematical reasoning, including the ability to analyze and evaluate problem situations, develop and evaluate mathematical arguments, use inductive and deductive reasoning, formulate generalizations, and draw contextual conclusions. Meanwhile, students with low ability demonstrated reasoning in information analysis, the use of deductive procedures, and the construction of arguments; however, they were not yet able to consistently formulate generalizations or draw conclusions within the context of the problem. This study has limitations because the gamified media used does not yet have zoom-in and zoom-out features, so in some parts, the display is still suboptimal when used by students on certain devices. Additionally, the learning and analysis of students' mathematical reasoning were still focused on the Pythagorean Theorem. Therefore, future research could be conducted on different mathematical topics. Further research could also focus on developing gamification media that are more interactive and adaptive to students' characteristics, thereby supporting the process of mathematical reasoning.

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