



## Hypothetical Learning Trajectory (HLT)-Based Learning Design Analysis

Rizka Fahuza Siregar<sup>1</sup>, Rani Rahim<sup>2\*</sup>, Rosmita Sari Siregar<sup>3</sup>, Oktaviana Nirmala Purba<sup>4</sup>

<sup>1</sup>Pembinaan Masyarakat Indonesia University, Medan, Indonesia

<sup>2</sup>Dharmawangsa University, Medan, Indonesia

<sup>3</sup>Prima Indonesia University, Medan, Indonesia

<sup>4</sup>Asahan University, Medan, Indonesia

\*Corresponding Authors: [ranirahim@dharmawangsa.ac.id](mailto:ranirahim@dharmawangsa.ac.id)

*Submitted: 12-05-2026*

*Revised: 17-05-2026*

*Accepted: 20-05-2026*

*Published: 05-06-2026*

### ABSTRACT

Mathematics learning trajectory is one of the key concepts in student-centered mathematics learning design. This article aims to analyze and describe the development of Hypothetical Learning Trajectory (HLT) as a framework for designing effective mathematics learning at the Junior High School level. The method used is design research with a qualitative approach through three phases: preparation phase, experimentation phase, and retrospective analysis phase. Research subjects consisted of 32 seventh-grade students at SMP Negeri 37 Medan. Research instruments included observation sheets, concept understanding tests, and semi-structured interviews. The results showed that HLT-based learning significantly improved students' conceptual understanding of integer material, with average scores increasing from 58.4 to 81.7. In addition, student learning motivation increased as demonstrated through the level of active participation in learning activities. The conclusions of this study confirm that systematic and reflective HLT development has great potential as an innovative strategy in meaningful and student-centered mathematics learning.

Keywords: conceptual understanding; design research; hypothetical learning trajectory; learning trajectory; mathematics learning

### INTRODUCTION

Mathematics is a fundamental discipline that plays a central role in the advancement of science, technology, and human reasoning. Mathematical thinking skills are required not only in academic settings but also in solving everyday problems and supporting 21st-century competencies such as critical thinking, creativity, communication, and problem-solving abilities. However, mathematics education in Indonesia continues to face significant challenges, particularly regarding students' low conceptual understanding, problem-solving ability, and learning motivation.

Evidence from international assessments consistently indicates that Indonesian students' mathematical achievement remains below the international average. The Programme for International Student Assessment (PISA) 2022 reported that Indonesia ranked 70th out of 80 participating countries with an average mathematics score of 366, considerably lower than the OECD average of 472 (OECD, 2023). Similar findings were also reported in Trends in International Mathematics and Science Study (TIMSS), where Indonesian students demonstrated difficulties in reasoning, conceptual understanding, and applying mathematics to real-world situations (Mullis, 2020). These results indicate that

current mathematics instruction has not fully facilitated meaningful conceptual learning and higher-order mathematical thinking.

Several studies suggest that one of the primary causes of students' low mathematical achievement is the dominance of teacher-centered instruction, where learning activities focus mainly on procedural fluency and memorization rather than conceptual understanding and active knowledge construction (Hadi, 2019); (Retnawati H. D., 2018). In such instructional environments, students tend to become passive recipients of information and have limited opportunities to explore mathematical ideas independently. (Freudenthal, 1991) argued that mathematics should be viewed as a human activity in which students actively reinvent mathematical concepts through meaningful experiences. Similarly, (Gravemeijer, K., & Doorman, M, 1999) emphasized the importance of instructional designs that support students' progressive mathematization processes through contextual and reflective learning activities.

To overcome these problems, mathematics learning requires an instructional design framework capable of systematically accommodating students' thinking processes and conceptual development. One framework that has received considerable attention in mathematics education research is the Hypothetical Learning Trajectory (HLT) proposed by (Simon, 1995). HLT refers to a hypothetical description of students' learning pathways consisting of learning goals, learning activities, and predictions of students' thinking processes during instruction. HLT enables teachers and researchers to anticipate how students' understanding may develop throughout learning activities and to revise instructional decisions based on classroom evidence.

The concept of HLT emerged within the Mathematics Teaching Cycle framework introduced by (Simon, 1995) as a model of teacher decision-making in constructivist classrooms. According to (Simon, 1995), HLT consists of three interconnected components: (1) learning goals that define the intended mathematical understanding, (2) learning activities designed to support conceptual development, and (3) hypothetical learning processes describing predictions of how students' thinking evolves during instruction. (Clements, D.H., & Sarama, J, 2004) further explained that learning trajectories provide a developmental map that helps teachers align instructional activities with students' cognitive growth. The term "hypothetical" highlights that predictions regarding students' learning are provisional and subject to revision through classroom observation and reflection.

The theoretical foundation of HLT is strongly rooted in constructivist and sociocultural learning theories. From a constructivist perspective, learning occurs when students actively construct knowledge through interaction with meaningful tasks and prior cognitive structures. Meanwhile, Vygotsky's sociocultural theory emphasizes the importance of social interaction, scaffolding, and the Zone of Proximal Development (ZPD) in facilitating conceptual growth (Vygotsky, 1978). Consequently, HLT-based instruction encourages students to engage in exploration, discussion, reflection, and collaborative problem-solving activities that promote deeper conceptual understanding (Bakker, 2018); (Stephan, M., & Akyuz, D, 2012).

Within the framework of design research or design-based research, HLT functions as an important instrument for transforming theoretical assumptions about learning into

practical instructional sequences (Gravemeijer, K., & Cobb, P, 2006). Design research involves iterative cycles of instructional design, classroom implementation, retrospective analysis, and redesign aimed at generating both effective learning products and theoretical contributions to educational practice (Cobb, 2003). In this process, HLT serves not only as a planning tool but also as an analytical framework for comparing predicted learning pathways with actual student learning processes observed during instruction (Confrey, J., & Maloney, A, 2015).

In recent years, HLT has been widely implemented within the framework of Realistic Mathematics Education (RME), which emphasizes the use of contextual and meaningful problems to support students' progressive mathematization (Gravemeijer, 1994); (van den Heuvel-Panhuizen, 2020). RME-based HLT designs have demonstrated effectiveness in improving conceptual understanding, reasoning ability, and student engagement across various mathematical topics and educational levels (Laurens, 2018); (Prahmana, 2021). In Indonesia, the adaptation of RME through Pendidikan Matematika Realistik Indonesia (PMRI) has encouraged the development of HLT-based instructional materials aligned with students' cultural and contextual experiences (Sembiring, 2010); (Zulkardi, 2020).

Empirical studies over the past decade have shown that HLT-based learning contributes positively to mathematics learning outcomes. Research by (Bakker, 2018) found that HLT supports the development of students' mathematical reasoning by enabling teachers to anticipate students' cognitive responses during instruction. Studies conducted by (Wijaya, 2020) and (Nusantara, 2021) demonstrated that HLT-based learning significantly improved students' conceptual understanding and problem-solving skills in junior secondary mathematics. In Indonesia, (Simorangkir, 2023) reported that learning activities developed through a Learning Implementation Trajectory (LIT) based on the PACE model effectively improved students' mathematical problem-solving abilities. Similarly, research by (Putra, 2022) showed that HLT integrated with PMRI principles enhanced students' participation and mathematical communication skills.

Despite the growing body of research on HLT, several gaps remain evident in the literature. First, many previous studies primarily focused on the effectiveness of HLT in improving learning outcomes, while limited attention has been given to analyzing the detailed process of developing HLT for specific mathematical topics, particularly integer concepts at the junior high school level. Second, previous studies generally emphasized learning outcomes quantitatively, whereas fewer studies explored how students' conceptual understanding develops throughout the learning trajectory process. Third, studies examining students' responses and engagement toward HLT-based learning in Indonesian classrooms remain limited, especially within the context of grade VII integer learning. Therefore, further research is needed to develop and analyze HLT designs that are contextually relevant and theoretically grounded in Indonesian mathematics classrooms.

Based on these gaps, this study offers several contributions and novelties. First, this research develops a context-based HLT specifically designed for integer material in grade VII junior high school students. Second, the study integrates constructivist principles, PMRI approaches, and design research methodology to produce a systematic and empirically validated learning trajectory. Third, this study not only evaluates learning outcomes but also

analyzes students' conceptual development processes and responses during HLT implementation. Thus, the study contributes both practically to the development of innovative instructional designs and theoretically to the enrichment of HLT research in mathematics education.

A learning trajectory refers to a description of the cognitive pathway students are expected to follow in understanding mathematical concepts. According to (Clements, D.H., & Sarama, J, 2004), a learning trajectory consists of three essential components: (1) clear learning objectives, (2) developmental progressions describing stages of student thinking, and (3) instructional activities designed to support conceptual growth. Unlike conventional linear instruction, learning trajectories acknowledge that students may develop mathematical understanding through diverse pathways and experiences (van den Heuvel-Panhuizen, 2020). Therefore, teachers function as facilitators who guide students through meaningful learning experiences rather than merely transmitting procedural knowledge.

Based on the background and research gaps described above, the research problems in this study are formulated as follows: (1) How is the process of developing a Hypothetical Learning Trajectory (HLT) for integer material in grade VII junior high school?, (2) How does HLT-based learning influence students' conceptual understanding of integers?, (3) How do students respond to mathematics learning designed using HLT principles?

Accordingly, this study aims to: (1) Describe systematically the process of developing an HLT for integer material in grade VII junior high school, (2) Analyze the influence of HLT-based learning on students' conceptual understanding of integers, (3) Identify students' responses and factors influencing the successful implementation of HLT-based mathematics learning.

## **RESEARCH METHODS**

This study employed a qualitative design research methodology consisting of three interrelated phases: preparation, design experiment, and retrospective analysis. The study was conducted at SMP Negeri 37 Medan during the odd semester of the 2025/2026 academic year.

The population of this study comprised all seventh-grade students at the school, while the sample consisted of 32 students from class VII-B, including 15 male and 17 female students. The sample was selected using purposive sampling based on the teacher's willingness to collaborate and the suitability of the class for the implementation of the research design. In addition, a pilot experiment involving six students with heterogeneous academic abilities was conducted prior to the main teaching experiment.

The research was carried out in three phases. The first phase was the preparation phase, in which the researchers designed the initial Hypothetical Learning Trajectory (HLT), prepared learning materials, and developed research instruments. The initial HLT and instruments were validated by two mathematics education experts and one experienced mathematics teacher to ensure content validity and suitability for classroom implementation.

The second phase was the design experiment phase, consisting of a pilot experiment and a teaching experiment. The pilot experiment was conducted with six students to evaluate the clarity and practicality of the learning activities. After revisions were made, the teaching

experiment was implemented with 32 students in class VII-B. During the learning process, the researcher acted as both teacher and observer to document students' learning trajectories, interactions, and problem-solving strategies.

The third phase was the retrospective analysis phase. In this phase, the researchers analyzed all collected data to compare the designed HLT with the actual learning process and to identify students' conceptual understanding of integers throughout the learning activities.

Data were collected using four instruments: (1) a conceptual understanding test consisting of 20 structured essay questions, (2) structured observation sheets to record classroom interactions and student activities, (3) semi-structured interview guidelines used with eight purposively selected students, and (4) documentation in the form of photographs, video recordings, and students' written work. Instrument validation was conducted through expert judgment involving mathematics education specialists and classroom practitioners.

Data analysis employed qualitative and descriptive statistical techniques. Qualitative data from interviews, observations, and documentation were analyzed through data reduction, coding, categorization, and interpretation to identify patterns in students' thinking processes. Quantitative data from the conceptual understanding test were analyzed descriptively by calculating mean scores, standard deviations, and percentages of learning mastery. Data analysis was conducted using Microsoft Excel for descriptive statistical calculations, while qualitative data were analyzed manually through thematic coding. Data validity was ensured through source triangulation and method triangulation.

## RESULTS AND DISCUSSION

### Development of Hypothetical Learning Trajectory (HLT)

Based on the literature review and needs analysis conducted during the preliminary phase, a Hypothetical Learning Trajectory (HLT) for integer material was developed for four learning sessions with a total allocation of 8 teaching hours ( $8 \times 40$  minutes). The HLT was designed to facilitate the gradual development of students' conceptual understanding from informal reasoning toward formal mathematical abstraction.

Table 1. Structure of the Hypothetical Learning Trajectory for Integer Learning

Session	Learning Focus	Learning Context	Predicted Student Thinking	Teacher Support
1	Understanding positive and negative integers	Temperature and altitude	Students identify positive and negative values from real situations	Guiding questions and concrete media
2	Addition and subtraction of integers	Interactive number line	Students use movement strategies on number lines	Visual scaffolding and peer discussion
3	Pattern generalization	Numerical patterns and contextual problems	Students connect informal strategies to symbolic forms	Class discussion and probing questions
4	Formalization of operations	Problem-solving activities	Students explain procedures independently	Reflection and mathematical justification

Table 1 illustrates the sequential learning trajectory developed to support students' cognitive progression. The learning design began with realistic contexts familiar to students before gradually introducing symbolic reasoning and formal procedures. This sequence reflects the principles of Realistic Mathematics Education (RME), which emphasize the importance of contextual learning experiences.

The developed HLT identified three major stages of cognitive development. The first stage was intuitive understanding, where students interpreted integers through meaningful contexts such as temperature, financial transactions, and elevation. At this stage, students generally recognized positive and negative numbers correctly, although many still experienced misconceptions when performing subtraction involving negative numbers.

The second stage was semi-formal understanding, in which students relied on visual representations such as number lines and concrete models to bridge informal reasoning and symbolic operations. During this stage, students increasingly demonstrated procedural confidence, although teacher scaffolding remained necessary.

The third stage was formal understanding, where students were able to perform integer operations independently and justify their reasoning symbolically without visual support.

### Implementation of HLT-Based Learning

The implementation of learning activities followed the predicted thinking pathways outlined in the HLT. In the first session, students explored integer concepts through temperature simulations using a giant thermometer model. This contextual activity successfully increased student engagement and curiosity, with 87.5% of students actively participating during classroom discussions.

In the second session, students investigated addition and subtraction operations using interactive number lines. Classroom observations revealed multiple problem-solving strategies. Some students used directional movement strategies ("*jumping right*" and "*jumping left*"), while others developed informal grouping and separating strategies.

An important finding during implementation was the emergence of unpredicted compensatory strategies. Several students decomposed numbers into simpler forms to simplify calculations. This finding demonstrates that students' actual learning trajectories may differ from the hypothetical trajectory initially designed.

Table 2. Students' Thinking Strategies During Integer Operations

Strategy Type	Description	Percentage of Students
Number line movement	Moving left/right on number line	43.8%
Grouping/separating	Combining or separating quantities	28.1%
Compensatory strategy	Decomposing numbers for easier calculation	18.7%
Symbolic algorithm	Direct symbolic manipulation	9.4%

Table 2 demonstrates that students used diverse reasoning strategies when solving integer problems. The dominance of visual and contextual strategies indicates that conceptual understanding developed before procedural fluency. The emergence of compensatory strategies also suggests that students actively constructed mathematical meaning rather than merely memorizing procedures.

In the third and fourth sessions, students gradually generalized their informal reasoning into formal operational rules. Through guided classroom discussions, students articulated mathematical patterns and connected them to symbolic notation.

These findings support (Gravemeijer, 1994) theory that HLT should be viewed as a flexible hypothesis rather than a rigid instructional sequence. The discrepancies between predicted and actual learning trajectories became valuable sources of pedagogical insight.

### Conceptual Understanding Test Results

Quantitative analysis showed a substantial improvement in students’ conceptual understanding after the implementation of HLT-based learning.

Table 3. Comparison of Pre-Test and Post-Test Results

Indicator	Pre-Test	Post-Test	Improvement
Mean score	58.4	81.7	23.3
Standard deviation	12.3	8.6	-3.7
Learning completion (%)	28.1%	87.5%	59.4%
Students achieving KKM	9 students	28 students	19 students

Table 3 indicates a notable increase in students’ conceptual understanding after the intervention. The increase in mean scores and completion rates suggests that HLT-based instruction effectively supported conceptual learning. In addition, the decrease in standard deviation indicates more homogeneous achievement among students after instruction.

Further analysis revealed that the greatest improvement occurred in students’ ability to interpret mathematical representations and apply concepts in contextual situations.

Table 4. Improvement in Conceptual Understanding Indicators

Conceptual Understanding Indicator	Improvement
Interpretation of representations	31.2
Application of concepts in context	28.7
Procedural understanding	19.4

The findings indicate that contextual and visual learning activities particularly strengthened students’ representational understanding. This supports the argument that meaningful contexts help students connect mathematical symbols with real-world situations. The results are consistent with previous studies showing that HLT-based learning improves conceptual understanding more effectively than conventional approaches. Research by (Clements, 2020) emphasized that carefully designed learning trajectories support students in constructing mathematical concepts gradually and meaningfully. Similarly, (Dougherty, 2023) found that integrating HLT with adaptive instruction enhances mathematical reasoning and classroom engagement.

The findings also align with studies conducted in Indonesia. (Fitriani, 2023) reported that HLT integrated with contextual PMRI activities improved students’ conceptual understanding in social arithmetic learning. Likewise, (Atiqoh, 2025) found that HLT-based

instruction accommodated differences in students' initial abilities and promoted deeper understanding of mathematical concepts.

However, the present study extends previous research by demonstrating that students can spontaneously develop alternative strategies beyond the predicted trajectory. This finding highlights the dynamic nature of mathematical learning and emphasizes the importance of adaptive teaching.

### **Supporting and Inhibiting Factors**

Several supporting factors contributed to the successful implementation of HLT-based learning:

1. The use of contexts closely related to students' daily experiences.
2. Teacher flexibility in adapting instruction based on students' responses.
3. A classroom environment encouraging exploration and discussion.
4. Activities designed within students' zone of proximal development.

Nevertheless, several inhibiting factors were also identified:

1. Limited instructional time reduced opportunities for deeper exploration.
2. Variations in students' prior knowledge complicated differentiated instruction.
3. Students initially struggled to adapt to student-centered learning because they were accustomed to teacher-centered instruction.

These challenges indicate that successful implementation of HLT requires careful classroom management and sufficient time allocation.

The findings of this study demonstrate that the implementation of HLT-based learning significantly improved students' conceptual understanding of integers. The improvement can be explained by the use of meaningful contexts and progressive representations that enabled students to construct mathematical ideas gradually. This supports the perspective of (Freudenthal, 1991) through the theory of Realistic Mathematics Education (RME), which emphasizes that mathematics learning should begin from contexts that are meaningful to students.

The successful use of temperature and financial transaction contexts indicates that contextual learning helps students connect abstract integer concepts with everyday experiences. Students were able to move from informal reasoning toward symbolic mathematical operations because the learning process provided opportunities for exploration and discussion. These findings are consistent with studies by (Clements, D.H., & Sarama, J, 2004), who emphasized that carefully designed learning trajectories can improve conceptual understanding more effectively than conventional instruction.

Another important finding was the emergence of unexpected student strategies during learning activities. This result supports the argument of (Gravemeijer, 1994) that HLT should be viewed as a flexible hypothesis rather than a fixed instructional script. The discrepancy between the predicted and actual learning trajectories reflects the dynamic nature of classroom learning. Instead of being considered a weakness, such discrepancies provide valuable insights for revising and improving instructional design.

The emergence of compensatory strategies also extends previous research regarding students' mathematical thinking processes. Earlier studies primarily focused on

predetermined learning paths, whereas this study demonstrates that students may generate spontaneous and creative reasoning strategies beyond teacher predictions. This finding highlights the importance of adaptive teaching competencies, particularly teachers' ability to interpret students' thinking and respond appropriately during classroom interactions.

The findings of this study are also related to previous research conducted by (Retnawati C. A., 2025), which reported that a Problem-Based Learning (PBL) model showed strong validity and practicality but had limited effectiveness in improving numeracy skills across different school categories. In contrast, the present study found substantial improvement in students' conceptual understanding after implementing HLT-based learning. This difference may be attributed to the structured prediction of students' thinking trajectories in HLT, which allows learning activities to better align with students' cognitive development stages.

From a practical perspective, the results imply that HLT-based learning can serve as an effective alternative approach for teaching abstract mathematical concepts, particularly integers. Teachers are encouraged to design learning trajectories that incorporate contextual problems, visual representations, and opportunities for student exploration. Moreover, teacher flexibility in adapting instruction based on students' responses is essential for maximizing learning effectiveness.

Despite these positive findings, several limitations should be acknowledged. First, the study involved a relatively small sample size and was conducted in a single school context, limiting the generalizability of the findings. Second, the implementation period was relatively short, consisting of only four learning sessions. Longer implementation periods may provide deeper insights into students' long-term conceptual development. Third, differences in students' prior knowledge and learning habits may have influenced the effectiveness of the intervention.

Future studies are recommended to involve larger participant groups across different educational contexts and investigate the long-term impact of HLT-based learning on mathematical reasoning, problem-solving ability, and student motivation.

## **CONCLUSION**

Based on the research results and discussions that have been described, several conclusions can be drawn as follows: First, the development of HLT for integer material in grade VII of junior high school was successfully carried out through a systematic and reflective process, resulting in three main stages of learning trajectories: context-based intuitive understanding, visual representation-based semi-formal understanding, and mathematical abstraction-based formal understanding.

Second, the implementation of HLT-based learning significantly improved students' understanding of integer concepts, as indicated by an increase in the average score from 58.4 to 81.7 and an increase in the percentage of learning completion from 28.1% to 87.5%. A greater increase occurred in conceptual understanding than in procedural understanding.

Third, the implementation of HLT in learning practices does not always go according to initial predictions, as students often develop unique and unexpected thinking strategies.

This requires teachers to be highly adaptive in responding to actual learning trajectories that occur in real time.

Fourth, key factors for the success of HLT implementation include the relevance of the context used, the suitability of the activity's challenge level to students' abilities, a classroom atmosphere that supports exploration, and teacher flexibility in adapting learning plans.

## REFERENCES

- Atiqoh, M. I. (2025). Hypothetical Learning Trajectory Based on Realistic Mathematics Education in SPLDV Material: Implementation to Accommodate Differences in Junior High School Students' Initial Abilities. *MATHEMA: Jurnal Pendidikan Matematika*, 8(1).
- Bakker, A. (2018). *Design Research in Education: A Practical Guide for Early Career Researchers*. Routledge.
- Clements, D. H. (2020). *Learning and Teaching Early Math: The Learning Trajectories Approach (2nd ed.)*. Routledge.
- Clements, D.H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, 6(2), 81–89. doi:[https://doi.org/10.1207/s15327833mtl0602\\_1](https://doi.org/10.1207/s15327833mtl0602_1)
- Clements, D.H., & Sarama, J. (2009). *Learning and Teaching Early Mathematics: The Learning Trajectories Approach*. Routledge.
- Cobb, P. C. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13.
- Confrey, J., & Maloney, A. (2015). A learning trajectory approach to instructional design. *ZDM Mathematics Education*, 47(1), 1–14.
- Dougherty, B. J. (2023). Integrating hypothetical learning trajectories with lesson study: Enhancing teaching practices. *Mathematics Teacher Educator*, 11(2), 134–152.
- Fitriani, S. D. (2023). Hypothetical Learning Trajectory Materi Aritmatika Sosial Berbantuan Video Interaktif: Konteks Jual Beli di Mal. *FIBONACCI: Jurnal Pendidikan Matematika dan Matematika*, 9(2).
- Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures*. Kluwer Academic Publishers.
- Gravemeijer, K. (1994). *Developing Realistic Mathematics Education*. CD-Beta Press.
- Gravemeijer, K., & Cobb, P. (2006). *Design research from the learning design perspective*. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.). (E. D. 17–51), Ed.) Routledge.
- Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics*, 39(1-3), 111–129. doi:<https://doi.org/10.1023/A:1003749919816>
- Hadi, S. &. (2019). TIMSS Indonesia: Students' mathematical reasoning ability. *Journal on Mathematics Education*, 10(2), 123–136.
- Laurens, T. B. (2018). How does realistic mathematics education improve students' mathematics cognitive achievement? *EURASIA Journal of Mathematics, Science and Technology Education*, 14(2), 569–578.
- Mullis, I. V. (2020). *TIMSS 2019 International Results in Mathematics and Science*. TIMSS & PIRLS International Study Center.
- Nusantara, T. S. (2021). Students' conceptual understanding through hypothetical learning trajectory. *Journal on Mathematics Education*, 12(3), 421–438.

- OECD, P. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*. OECD Publishing. doi:<https://doi.org/10.1787/53f23881-en>
- Prahmana, R. C. (2021). Design research in mathematics education: A systematic literature review. *Journal on Mathematics Education*, 12(3), 395–414.
- Putra, Z. H. (2022). PMRI-based learning trajectory in mathematics instruction. *Infinity Journal*, 11(1), 45–58.
- Retnawati, C. A. (2025). Development of A Problem-Based Learning Model to Improve Students' Numeracy and Self-Efficacy. *Prisma*, 14(2), 228-241.
- Retnawati, H. D. (2018). Teachers' difficulties in implementing 21st-century learning. *International Journal of Instruction*, 11(3), 1-16.
- Sembiring, R. K. (2010). Reforming mathematics learning in Indonesian classrooms through PMRI. *ZDM Mathematics Education*, 42(7), 791–804.
- Simon, M. A. (1995). Reconstructing mathematical pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26(2), 114–145. doi:<https://doi.org/10.2307/749205>
- Simon, M.A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, 6(2), 91–104. doi:[https://doi.org/10.1207/s15327833mtl0602\\_2](https://doi.org/10.1207/s15327833mtl0602_2)
- Simorangkir, F. M. (2023). Pengembangan Local Instruction Theory Berbasis Model Pace untuk Melatihkan Kemampuan Pemecahan Masalah Matematis Siswa. *PRISMA*, 12(2), 370–379. doi:<https://doi.org/10.35194/jp.v12i2.3267>
- Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal of Mathematical Behavior*, 31(4), 428–464.
- Treffers, A. (1987). *Three Dimensions: A Model of Goal and Theory Description in Mathematics Instruction—The Wiskobas Project*. D. Reidel Publishing.
- van den Heuvel-Panhuizen, M. &. (2020). Realistic Mathematics Education. *In Encyclopedia of Mathematics Education*.
- Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics*, 54(1), 9–35. doi:<https://doi.org/10.1023/B:EDUC.0000005212.03219>
- Vygotsky, L. S. (1978). *Mind in Society*. Harvard University Press.
- Wijaya, A. R. (2020). The role of hypothetical learning trajectory in improving conceptual understanding. *Journal on Mathematics Education*, 11(2), 287–302.
- Zulkardi, Z. &. (2020). PMRI and Indonesian mathematics education reform. *Journal on Mathematics Education*, 11(1), 1–16.