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Pre-Service Mathematics Teachers Learn the Concept of Conic Sections: A Praxeological Analysis of Textbooks

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ABSTRACT

This study examines how conic sections are represented and transposed in Indonesian high school mathematics textbooks used in the *Kapita Selekta Matematika Sekolah Lanjutan* course for pre-service mathematics teachers (PSMTs), using the Praxeological-Didactical Analysis (PDA) framework as its analytical lens. This study identifies a structural imbalance at the textbooks analyzed: the *logos* block that layer of mathematical knowledge responsible for explaining and justifying techniquesmis largely absent. By mapping the full T-τ-θ-Θ structure across textbook tasks, the findings show how an overreliance on algebraic manipulation constrains PSMTs' ability to explain not just *how* a technique works, but *why* it is mathematically valid. In response, the study proposes a reconstructed praxeology-based task design model that deliberately integrates technological and theoretical articulation into instructional sequences. Beyond its immediate findings, this work reframes what PDA can do: not merely as a tool for diagnosing curricular problems, but as a generative framework for rebuilding knowledge transposition in mathematics teacher education.

Keywords: praxeology; textbook analysis; pre-service mathematics teachers; conic sections; didactical transposition

INTRODUCTION

Pre-service mathematics teachers (PSMTs) carry a particular kind of responsibility. They are not simply learning mathematics for themselves; they are preparing to make mathematics accessible and meaningful to others. This means that understanding a procedure is never quite enough. They need to understand why the procedure works, where it comes from, and how to explain it to a student who has never seen it before. And yet, research consistently shows that this deeper layer of understanding is precisely what many PSMTs lack, especially in intersection of geometric and algebraic thinking. Conic sections are one such topic. Sadidah and Sudihartinih (2023) found that more than half of PSMTs are struggling with parabolas, misunderstanding ellipses, and in many cases being unable to engage with hyperbolas at all. Advíncula Clemente et al. (2021) show that even experienced teachers retain only limited conceptual understanding of these curves, suggesting that whatever gaps form during pre-service preparation tend to follow teachers into their classrooms.

This raises an uncomfortable question: if the problem is this persistent, what is sustaining it? Part of the answer, this study argues, lies not in individual learners but in the materials that shape their learning. In Indonesia, 93% of teachers depend on textbooks as their main instructional resource (Human Development Sector, 2010), and textbooks have long been understood to influence not only what gets taught, but how teachers think about teaching it (Public First, 2021). For PSMTs specifically, textbooks do something more than provide information; they model what mathematics is supposed to look like. They show future teachers how concepts should be introduced, what counts as a sufficient explanation, and where the boundaries of understanding are expected to lie. When those models are impoverished, the consequences extend well beyond the textbook itself.

The deeper issue is that textbooks do not simply transmit mathematical knowledge; they transform it. This is the central insight behind Chevallard's (1985, 1999) concept of didactical transposition, developed within the Anthropological Theory of the Didactic (ATD). As mathematical knowledge moves from the research community to curriculum documents to textbooks and finally into classrooms, it undergoes a series of adaptations at each stage. Some of these adaptations are necessary and even productive. But others quietly erode what made the knowledge meaningful in the first place. When the pressure to make content teachable leads to the removal of justifications, the suppression of geometric origins, and the reduction of rich mathematical objects to sets of manipulable symbols, the result is a version of mathematics that can be performed but not truly understood. Chevallard (1999) calls the cognitive consequences of this process epistemological obstacles; not simply gaps in knowledge, but institutionally embedded distortions that actively prevent learners from developing a coherent understanding of the subject. These are not mistakes that better students could avoid. They are structural features of how knowledge has been organized and presented to them.

Conic sections are particularly susceptible to this kind of distortion. The topic is inherently relational; it asks learners to hold geometric definitions and algebraic representations in mind simultaneously, to understand that a parabola is not merely an equation but a set of points with a precise geometric meaning. Yet in many Indonesian textbooks, that geometric grounding is bypassed almost immediately. The equation appears first; the derivation, if it appears at all, is presented as a formality. PSMTs learn to substitute values, complete the square, and identify standard forms; and they do so without ever being asked to explain why those forms arise or what the underlying geometry is doing. The technique is taught. The justification for the technique is not. And a PSMT who cannot justify a technique cannot teach it. They can demonstrate it, but they cannot respond when a student asks why.

What is needed is a framework that can make this structural problem visible. Praxeological-Didactical Analysis (PDA), rooted in ATD, does exactly that. Rather than asking only what students are expected to do, PDA asks whether the materials give them any reason to do it. By mapping mathematical knowledge across four levels: tasks (T), techniques (τ), technologies (θ), and theories (Θ); PDA allows us to see not just the procedural surface of a textbook, but the presence or absence of the conceptual and theoretical architecture beneath it. Crucially, the absence of *logos* components that are the technologies and theories that justify and explain techniques is not treated as a minor oversight. Within ATD, it is a meaningful structural feature, one with direct consequences for how PSMTs come to understand and eventually teach the mathematics they have encountered.

Previous praxeological studies in Indonesian mathematics education have examined functions (Utami et al., 2024), proportional reasoning (Wijayanti & Winsløw, 2017), and rational numbers (Putra, 2020), among other topics. None have turned this lens on conic sections. But the gap here is not merely about which topic has or has not been studied. The more significant absence in the literature is theoretical: no existing study asks how the praxeological organization of conic sections in textbooks might be actively generating the learning difficulties that researchers have spent years documenting. Prior work has described what PSMTs struggle with. This study asks what the textbooks they learned from may have done or failed to do to produce those struggles. The reintroduction of conic sections into the Indonesian curriculum in 2013, after nearly a decade of absence, makes this question more urgent. With no established pedagogical tradition, PSMTs and teachers alike have been left to reconstruct their understanding of the topic through whatever textbooks place in front of them. If those textbooks are structurally incomplete, they show up directly in how the next generation of teachers understands and teaches mathematics.

This study therefore takes up PDA a way of asking hard questions about what textbooks are actually doing when they present conic sections to future teachers. Specifically, it examines: (1) the types of tasks (T) that appear in conic section materials; (2) the techniques (τ) those tasks call for; (3) the technologies (θ) when the conceptual explanations give those techniques their meaning; and (4) the theories (Θ) that form the mathematical foundation of the topic. The aim is not simply to catalogue what is present, but to understand what is consistently missing and to reckon PSMTs who are being prepared not just to solve problems, but to help others understand them.

RESEARCH METHODS

This study employed a qualitative research design with a document analysis approach. The purpose of this design was to examine in detail how PSMTs learn mathematical concepts specifically conic sections through the structure of tasks presented in mathematics textbooks. Document analysis was chosen because it allows the systematic investigation of written materials to identify patterns, meanings, and implicit assumptions related to mathematical practices (Bowen, 2009). In this context, textbooks were not treated as sources of exercises but as cultural and didactical artifacts that reflect how mathematical knowledge is organized, justified, and transposed for instructional purposes.

The analysis followed the framework of praxeological-didactical analysis (PDA), which is rooted in the anthropological theory of the didactic (Chevallard, 1999, 2007). This framework considers mathematical activity as a system composed of four interrelated components: tasks (T), techniques (τ), technology (θ), and theory (Θ). Each component represents a layer of mathematical practice from concrete actions to abstract reasoning that shapes how mathematical knowledge is transmitted and learned. By examining the presence and balance of these components in textbook materials, the study uncovers how praxeological structures may support or hinder the development of conceptual understanding among PSMTs.

The data sources in this study consisted of senior high school mathematics textbooks used in the *Kapita Selekta Matematika Sekolah Lanjutan* course at teacher education institutions (LPTKs) in Indonesia. The selected textbooks covered the topic of conic sections, including parabolas, ellipses, and hyperbolas. Each task related to these topics was identified, categorized, and analyzed based on its praxeological structure. The classification involved mapping tasks (T) according to their mathematical objectives, identifying the techniques (τ) to solve them, and determining the technological (θ) and theoretical (Θ) explanations were provided.

The analytical process proceeded in three main stages. First, the task identification stage involved extracting all exercises and examples related to conic sections from the selected textbooks. Each task was coded according to its type, representation (symbolic, visual, or verbal), and complexity level. Second, the praxeological decomposition stage examined each task to determine the corresponding technique (τ), the presence or absence of explicit technological justification (θ), and the underlying theoretical reference (Θ). Finally, the interpretive stage involved synthesizing the findings to reveal dominant patterns, epistemological gaps, and potential learning obstacles that may arise from the textbook structures. Through this process, the analysis aimed to balance or imbalance the components of praxeology reflect the transposition of mathematical knowledge from scholarly to taught knowledge for student.

The researcher served as the primary instrument in this qualitative document analysis, taking responsibility for interpreting textual data, conducting iterative readings, and systematically applying the Praxeological Didactical Analysis (PDA) framework. As the main analytic instrument, the researcher engaged in identifying tasks, techniques, technologies, and theories, while monitoring the influence of personal assumptions throughout the analytic process. Methodological rigor was ensured through triangulation, conducted by comparing multiple textbooks from different publishers and validating interpretations against theoretical constructs from the literature. Reflexive notes, research logs, and analytic memos were maintained to document decision making. To minimize interpretive bias, the researcher consulted with an expert in mathematics education to discuss preliminary coding, praxeological categorizations, and ambiguous cases, strengthening the trustworthiness of the findings. By combining the principles of document analysis with the theoretical depth of PDA, this study provides a comprehensive picture of how textbooks mediate the learning of conic sections for PSMTs and offers insights that may inform the development of mathematics textbooks and didactical approaches.

RESULTS AND DISCUSSION

Classification of Task Types in the Textbook

In this study, the initial analysis focused on six main types of tasks (T1 to T6) identified in the mathematics textbooks used by PSMTs during the Kapita Selekta Matematika Sekolah Lanjutan course. These textbook tasks, which appear as worked examples and exercises, serve as an essential entry point for understanding the mathematical activities of PSMTs. They are relevant because the tasks represent the foundational problems when learning the topic of conic sections.

As summarized in Table 1, PSMTs were required to complete 11 tasks, categorized into six distinct types, each designed to support their conceptual understanding of conic sections. Task Type T1, required PSMTs to construct the equation of a parabola based on given geometric information, such as the vertex, focus, opening direction, or directrix. This type of task aims to strengthen students' ability to translate between geometric properties and algebraic representations, which is a crucial step in building a coherent understanding of conic sections.

Furthermore, Task Type T1, as a form of algebraic concept, contributes to conceptual clarity in geometry by fostering the coordination between symbolic reasoning and spatial visualization. This finding aligns with Paul and Sahidullah (2025), who argue that integrating algebraic formulation with geometric reasoning enhances students' spatial awareness and improves their efficiency in mathematical problem solving.

Table 1. Task Types and Examples for Introducing Conic Sections

Task Types (T)	Tasks (t)
T_1: Constructing the equation of a parabola from geometric information	$T_{1,1}$: If a parabola has a vertex at (2,3) and focus at (1,3), determine its equation. $T_{1,2}$: Given a parabola whose axis of symmetry is the line $y = 1$ and the distance between the focus and the directrix is 2. If the parabola's vertex lies on the line $x - y = 1$ and the parabola opens to the right, find its equation!
T_2: Identifying geometric elements from a parabola's equation	$T_{2,1}$: Consider the parabola defined by the equation $y^2 - 6y - 20x - 31 = 0$. Determine the parabola's center and vertex!
T_3: Constructing the equation of an ellipse from geometric information	$T_{3,1}$: Find the equation of an ellipse with center O(0,0), focus at (4,0) and (−4,0), and a major axis length of 12! $T_{3,2}$: Determine the equation of an ellipse with center (1,−2), given that one focus is at $(1,1\frac{1}{2})$ and the minor axis length is 1! $T_{3,3}$: Find the equation of an ellipse whose foci are at (−2,1) and (4,1) and vertices are at (−4,1) and (6,1)!
T_4: Identifying geometric elements from an ellipse's equation	$T_{4,1}$: Given the ellipse $9x^2 + 4y^2 = 36$, determine the following: center, foci, vertices, lengths of the major and minor axes, length of the latus rectum, equations of the directrices, and the eccentricity?
T_5: Identifying geometric elements from a hyperbola's equation	$T_{5,1}$: Determine the elements of the hyperbola, which consist of the center, foci, vertices, length of the transverse axis, and the equations of the asymptotes for the given hyperbola equation: a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$ b. $9y^2 - 16x^2 = 144$ $T_{5,2}$: For the hyperbola given by $9x^2 - 4y^2 - 18x - 24y = 26$, determine the center, foci, vertices, transverse axis length, and asymptotes.
T_6: Constructing the equation of a hyperbola from geometric information	$T_{6,1}$: Determine the equation of a hyperbola whose foci are at (8,0) and (−8,0) and vertices are at (5,0) and (−5,0)! $T_{6,2}$: Determine the equation of a hyperbola whose vertices are at (4,2) and (−2,2), given that one of its asymptotes is $2x - 3y + 4 = 0$!

Task Type T2, in contrast, challenges students to identify the geometric elements of a given parabola equation particularly when the equation is not written in its standard form. This requires students to understand how algebraic transformations work and to interpret the meaning of coefficients in relation to the parabola's geometry.

For the topic of ellipses, Task Types T3 and T4 emphasize two complementary directions of mathematical reasoning. Task Type T3 asks students to construct the equation of an ellipse based on geometric information such as the center, foci, and the lengths of the major and minor axes. The ability to apply the technique of completing the square becomes essential in solving such problems. Conversely, Task Type T4 requires students to extract geometric information from a given equation of an ellipse, including the coordinates of the foci, the center, the length of the latus rectum, and the eccentricity.

Task Types T5 and T6 are related to hyperbolas. Task Type T5 assesses students' ability to interpret the geometric properties of a hyperbola from its equation such as the center, vertices, foci, and asymptotes. The technique used typically involves manipulating quadratic forms and interpreting the general structure of the hyperbola's equation. Task Type T6, which shares structural similarities with T1 and T3, asks students to construct the equation of a hyperbola from given geometric data. Due to this similarity, it was later considered for integration with other construction-based task types in the analysis.

Collectively, these six task types represent two major representational pathways in learning conic sections: from geometry to algebra (T1, T3, T6) and from algebra to geometry (T2, T4, T5). These pathways are crucial because they reflect the didactical transposition that pre-service mathematics teachers should experience moving between visual and symbolic forms to build interconnected understanding. Within the pedagogical context of the *Kapita Selekta* course, mastering this representational flexibility is essential, as it forms the foundation for explaining conic section concepts meaningfully to future high school students.

The analysis also revealed that the textbook tends to emphasize procedural techniques and symbolic manipulation. This reflects what Sfard (1991) describes as the operational conception of mathematics, an understanding of mathematics as a set of procedures or actions. In contrast, the structural conception, which views mathematical concepts as interconnected objects, receives less attention. For instance, the relationships between parameters in the equation of an ellipse or between the gradient and the position of the asymptotes in a hyperbola are rarely made explicit in the worked examples or exercises presented in the textbook.

Therefore, although the local praxeology for each type of conic section is represented to some extent in the material, there remains significant room for improvement through the development of more exploratory and conceptually oriented task designs. In follow-up discussions with mathematicians and course instructors, it was agreed that introducing visually oriented task types such as sketching graphs from equations or deriving equations from given figures would provide richer learning experiences. Such tasks would help PSMTs not only perform symbolic manipulations but also understand and explain the meaningful connections between geometric visualization and algebraic form.

Accordingly, these initial findings provide a strong foundation for developing a more comprehensive and conceptually balanced praxeological organization for teaching and learning conic sections in teacher education programs. The results also serve as a basis for proposing alternative task types and recommending the strengthening of the technological and theoretical components that underpin mathematical practices in university-level instruction.

Techniques Used in the Textbook

Within the framework of didactical praxeological analysis, technique (τ) refers to the specific procedures or operational steps employed to solve a given mathematical task (T). The analysis of solution techniques presented in the textbook used for the Kapita Selekta Matematika Sekolah Lanjutan course reveals a range of strategies applied to address various types of conic section problems. However, the level of explicitness with which these techniques are introduced and explained varies considerably across topics.

Table 2. Techniques for Task Types Related to Introductory Functions

Task Type (T)	Technique (τ)	Description of technique
T_1	τ_1: Logical τ_2: Operational τ_3: Memorial	• Determine the orientation of the parabola by observing the position of the vertex and the focus. • Identify all the required parameters and formulate the standard form of the parabola. • Use the equation of the parabola $(y-n)^2 = 4p(x-m)$
T_2	τ_1: Logical τ_2: Algebraic τ_3: Operational τ_4: Sequential	• Determine the orientation of the parabola by examining the positions of the vertex and the focus. • Convert the equation into the standard form of a parabola by using the completing-the-square method. • Identify the elements of the parabola (vertex, focus, directrix, axis, etc.) from the resulting standard form. • Carry out a step-by-step procedure from the general equation to obtain all desired parabola elements.
T3	τ_1: Logical τ_2: Memorial τ_3: Operational	• Determine the orientation of the ellipse based on the given elements. • Use the formulas $c^2 = a^2 - b^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to find the parameters of the ellipse. • Identify the elements of the ellipse from the given geometric data.
T_4	τ_1: Logical τ_2: Algebraic τ_3: Operational τ_4: Sequential	• Determine the structural form of an ellipse by examining the parameters in a given ellipse equation. • Convert the general equation into the standard form of an ellipse by using the completing-the-square method. • Identify the elements of the ellipse (center, vertices, foci, major/minor axes, etc.) from the resulting standard form.

		• Perform a step-by-step procedure from the general equation to obtain all desired elements of the ellipse.
T$_5$	τ_1: Logical τ_2: Algebraic τ_3: Operational τ_4: Sequential τ_5: Memorial	• Determine the structural form of a hyperbola by examining the parameters in a given hyperbola equation. • Convert the general equation into the standard form of a hyperbola by using the completing-the-square method. • Identify the elements of the hyperbola and calculate the equations of its asymptotes. • Perform a step-by-step procedure from the given equation to obtain all the desired elements of the parabola. • Apply the formula for determining the asymptotes.
T$_6$	τ_1: Logical τ_2: Algebraic τ_3: Operational τ_4: Sequential	• Determine the opening direction of a hyperbola by examining the parameters in a given equation. • Rewrite the general equation into the standard form of a hyperbola by completing the square and performing the required algebraic manipulations. • Identify the elements of the hyperbola (center, vertices, foci, transverse/conjugate axes, etc.) from the resulting standard form. • Carry out a step-by-step procedure from the general equation to derive all required elements of the hyperbola.

To identify the techniques used in each task type on conic sections, this study began by examining the solution strategies presented in senior high school mathematics textbooks used by PSMTs in the *Kapita Selekta Matematika SMA Lanjutan* course. In this context, a technique is understood as the specific procedure or method employed to solve a particular type of task (Chevallard, 1999), whether explicitly demonstrated through worked examples or implicitly conveyed through recurring solution patterns in practice exercises.

The techniques identified in the textbooks were then classified into five main categories: operational, algebraic, sequential, logical, and memorial. These categories were adapted from several previous studies that explored mathematical activity and cognition. Tall (1996) categorized symbolic and procedural techniques as essential components of conceptual understanding in mathematics. Duval (1995) differentiated various forms of cognitive apprehension, including sequential and logical apprehension. Meanwhile, Solis and Isoda (2022), as well as Takeuchi and Shinno (2020), emphasized the significance of procedural and logical techniques in geometric problem solving, along with the role of memorized or memorial strategies commonly found in textbook exercises.

As summarized in Table 2, the techniques used to solve conic section tasks are predominantly operational and algebraic in nature. Operational techniques involve the use of formulas and parameter computations such as determining axis lengths, distances, or eccentricity values. These procedures frequently appear as direct substitutions based on the given information. Algebraic techniques, on the other hand, refer to symbolic manipulation processes, such as completing the square or converting a general quadratic form into the standard equation of a parabola, ellipse, or hyperbola.

In certain task types, such as T2, T4, and T6, sequential techniques also emerge. These techniques involve systematic transformations of symbolic expressions through a step-by-step process. For example, in Task Type T2 (transforming a quadratic equation into standard form), students are required to follow a specific sequence of algebraic operations to reach the final expression. Such techniques reflect the learner's ability to perform ordered procedures—an aspect that, according to Duval (1995), is central to sequential apprehension in mathematical cognition.

The logical technique emerges when students are required to determine the form of an equation based on geometric information that is not always explicitly stated, such as the opening direction of a parabola or the orientation of a hyperbola's axis. This type of reasoning, although not explicitly demanded in the task, occurs internally and reflects what Duval (1995) refers to as logical apprehension. For instance, in the task of constructing the equation of a hyperbola (T6), students must infer the algebraic form of the equation from the position of the focus and vertices, or by connecting the information about the asymptote line to the ratio of the parameters aa and bb.

Meanwhile, the memorial technique refers to strategies that rely primarily on recalling formulas without further exploration of their meaning or origin. This approach is commonly found in tasks such as T3 and T5, where students simply apply formulas $c^2 = a^2 - b^2$ or the equation of a hyperbola's asymptote to solve problems, without being encouraged to justify why these formulas hold true in the given context.

The analysis indicates that the textbook generally promotes algorithmic and procedural forms of reasoning specifically operational and algebraic techniques while techniques that foster logical reasoning and structural understanding (logical and sequential techniques) appear only in limited contexts and are not explicitly scaffolded. The frequent use of memorial techniques also suggests that task design tends to be result-oriented, prioritizing the final answer over the reasoning process.

To foster a deeper conceptual understanding among PSMTs, it is therefore necessary to develop task designs that explicitly activate logical reasoning and structured thought sequences. Furthermore, there should be a critical reflection on the predominance of memorization-based strategies to prevent students' understanding from remaining at a merely procedural level.

Overall, the techniques presented in the textbook are fairly representative of the conventional approach to teaching conic sections in high school mathematics. However, their presentation remains largely procedural, with minimal emphasis on strategy variation or flexibility in selecting alternative approaches. This limitation may restrict PSMTs ability to evaluate and compare alternative techniques and to develop didactical sensitivity toward the potential learning difficulties experienced by secondary students.

Hence, the analysis underscores the need to broaden the range of techniques featured in the textbook by including alternative strategies and emphasizing conceptual justification. Strengthening PSMTs reflective skills in selecting appropriate techniques for different contexts would make a substantial contribution to their pedagogical and professional competence as future mathematics educators.

Technological Discourse Underlying the Solution Techniques

Within the framework of didactical praxeology, technological discourse (θ) refers to the conceptual reasoning that explains or justifies the use of a particular technique (τ) in solving a mathematical task (T). Technology serves as the bridge between the procedures applied in practice and the formal theory (Θ) that underpins them. Consequently, the presence and quality of technological discourse are crucial in fostering meaningful understanding particularly for PSMTs, who are expected not only to demonstrate procedural fluency but also to articulate and transmit conceptual reasoning to their future students.

The analysis of the logos block was carried out to identify and compare the technological discourses (θ) found in the textbook with the formal definitions or theories (Θ) derived from relevant mathematical literature. In this context, technology is understood as a discourse about techniques that is, the way of thinking or reasoning that underlies the use of certain techniques in solving the analyzed task types (Bosch & Gascón, 2014; Chevallard, 2007).

Based on the analysis of the textbook, two main technological components (θ_1 and θ_2) were identified as the foundations of the local praxeologies represented in the topic of conic sections (parabolas, ellipses, and hyperbolas). These technological discourses were then compared with the formal theoretical references (Θ) from analytic geometry, as summarized in Table 3.

Table 3. Comparison between Textbook Technologies (θ) and Formal Theoretical Definitions (Θ)

Technology (θ)	Textbook's definition	Formal definition
θ_1: From the equation to the geometric elements.	Manipulating the general form of the equation into the standard form through the process of completing the square.	Identification of the geometric elements of a conic section equation is based on the theory of quadratic forms in two variables. This process involves transforming the general quadratic equation into its canonical form using algebraic techniques (such as completing the square).
θ_2: From the geometric elements to the equation.	• Identifying the main elements from the standard equation. • Constructing the conic section equation based on the given geometric data.	• The construction of a conic section equation from geometric conditions is based on the locus definition for each type of curve: A parabola is defined as the set of all points that are equidistant from a fixed point (focus) and a fixed line (directrix). An ellipse is the set of all points for which the sum of the distances to two fixed points (foci) is constant. • A hyperbola is the set of all points for which the difference of the distances to two fixed points (foci) is constant.

The first technological component (θ_1) relates to the activity of identifying the geometric elements of a conic section from its given equation. In the textbook, this approach is carried out by transforming the general quadratic form typically involving two variables into its standard form, often through the technique of completing the square. The main objective of this process is to obtain key geometric information such as the center, vertex, opening direction, axis lengths, and eccentricity. This technology operates primarily through symbolic procedures and algebraic manipulation; however, it does not always provide a conceptual explanation of the relationships between the algebraic form and its geometric meaning.

In formal theory, this activity falls within the domain of analytic geometry and the theory of quadratic forms, where algebraic transformations of general quadratic equations allow for geometric interpretations of conic properties. The process extends beyond symbolic manipulation, it embodies the deep relationship between symbolic representations and geometric objects, showing how equations encode spatial structure.

The second technological component (θ_2) refers to the construction of conic section equations from given geometric data, such as the position of the focus, vertex, directrix, or asymptote. In the textbook, this is typically introduced through the presentation of standard forms for each type of conic, followed by direct substitution of the known parameters. While this strategy is efficient in producing the expected equation, it provides limited opportunity for students to explore the underlying meaning of the curve's definition.

In contrast, formal mathematical theory grounds this activity in the locus definitions of conic sections. A parabola is defined as the set of all points equidistant from a fixed point and a line; an ellipse as the set of points for which the sum of distances to two fixed points (foci) is constant; and a hyperbola as the set of points for which the absolute difference of those distances is constant. These definitions are rooted in Euclidean geometry and are formalized within analytic geometry through the distance formula and coordinate representation. Hence, constructing equations from geometric elements, in the formal sense, requires a more conceptual understanding of spatial relationships rather than mere numerical substitution.

This logos block analysis thus reveals an epistemological gap between the technological discourse presented in the textbook and the formal theoretical framework. The textbook emphasizes procedural and symbolic aspects of problem solving but rarely makes explicit the conceptual principles underlying the algebraic forms or geometric properties of conic sections. This gap is particularly significant in the context of teacher education, as it may limit pre-service teachers' conceptual depth in understanding the connections between symbolic expressions and geometric structures. Bridging this gap through explicit articulation of conceptual reasoning could enhance both mathematical understanding and didactical awareness in future teachers.

Based on the analysis of the textbook used in the *Kapita Selekta Matematika SMA Lanjutan* course, the forms of conceptual reasoning that appear in the conic section materials particularly in the topic of parabolas vary in both their level of explicitness and depth. In general, the technological discourse presented in the textbook can be classified into two main categories:

1. Explicit technological discourse, referring to direct explanations embedded in the narrative or in worked examples. For instance, the textbook explicitly defines a parabola as the set of all points equidistant from a fixed point (focus) and a fixed line (directrix). This explanation typically appears in the introductory part of the topic and serves as the conceptual foundation for why certain algebraic forms of equations are used in the solution techniques. In some examples, the text also provides reasoning on why the canonical form of the equation is more convenient for geometric interpretation.

2. Implicit technological discourse, which refers to justifications that are not explicitly stated but are implied through the way techniques are presented or applied. For example, when solving a problem that involves constructing the equation of a parabola from a given focus and vertex, the direct use of the formula is not always accompanied by an explanation of why that formula is relevant. In such cases, students are expected to reconstruct the reasoning behind the chosen technique based on their prior understanding and intuition.

In addition, the relationship between technology and technique in the textbook tends to be local and dependent on individual examples. The book does not systematically connect the conceptual explanations across different examples or across different types of tasks. This lack of coherence can make it difficult for students to perceive the underlying unity among the techniques and the conceptual principles that justify them especially for those who have not yet developed strong reflective learning habits.

This fragmented technological discourse poses a challenge in teacher education such as how to PSMTs build bridges between procedural execution and the mathematical principles that justify those procedures. Therefore, it is essential for educators and textbook authors to design more explicit and systematic technological narratives that make the reasoning behind techniques transparent. By engaging with this type of discourse, pre-service mathematics teachers will not only learn how to perform techniques but also why they work, thereby strengthening their ability to develop didactical and pedagogical knowledge when teaching conic section concepts in secondary classrooms.

CONCLUSION

This study provides a praxeological analysis of how conic section concepts are presented in the mathematics textbook used in the *Kapita Selekta Matematika SMA Lanjutan* course. The findings show that while the textbook offers a relatively organized structure of tasks and techniques, its technological discourse (θ) and theoretical grounding (Θ) remain largely implicit. As a result, the learning trajectory supported by the textbook tends to emphasize procedural fluency over conceptual understanding.

The study concludes that the dominance of procedural techniques, combined with limited explanation of conceptual rationales, may inhibit PSMTs from constructing coherent connections between algebraic and geometric representations. This lack of representational integration can restrict the development of reflective and structural mathematical thinking that is essential for future teachers. Strengthening explicit technological explanations and reinforcing theoretical links are necessary to support deeper understanding of conic sections.

The analysis also highlights the need for broader and more varied task types that promote representational flexibility. The reconstructed task organization proposed in this study demonstrates how adding visualization, graph construction, and symbolic interpretation tasks can enrich the praxeological structure without introducing new theories or technologies. Such improvements align with the goals of teacher education to cultivate strong content knowledge and conceptual awareness in PSMTs.

Overall, the study emphasizes that meaningful learning of conic sections should extend beyond symbolic manipulation. An integration of visualization, conceptual explanation, and representational transition is essential to prepare PSMTs to teach conic sections in ways that foster genuine mathematical understanding among secondary school students.

Based on the findings of this study, several recommendations can be made to enhance the learning and teaching of conic sections in pre-service mathematics teacher education. Lecturers are encouraged to complement textbook instruction with explicit discussions on the technological reasoning (θ) that justifies solution techniques and the theoretical foundations (Θ) underlying conic section concepts. Integrating visualization activities, graph interpretation, and verbal-to-symbolic translation tasks into classroom practice can also support PSMTs in developing flexible representational understanding. Textbook authors and curriculum developers may consider incorporating a broader range of task types—such as those involving graph-based construction and interpretive visualization—to balance procedural and conceptual learning.

The results indicate the need to refine the praxeological organization of conic section tasks used in teacher education programs. The expanded set of task types proposed in this study can serve as a foundation for developing local didactical praxeologies that more effectively link techniques, technologies, and theories. Strengthening the explicit articulation of θ and Θ within instructional materials will also help reduce epistemological gaps and support deeper conceptual engagement with conic sections.

Further studies may examine how the proposed praxeological reorganization functions when implemented in classroom settings, particularly within didactical design research (DDR). Future research may also extend the praxeological analysis to other conic section topics or explore how PSMTs' understanding evolves after engaging with enriched representational tasks. Comparative studies involving different textbooks or instructional approaches would provide additional insights into the role of praxeology in shaping mathematical understanding.

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