



Students' Mathematical Connections in Solving Word Problems Based on Polya's Stages

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ABSTRACT

The purpose of this study was to describe the mathematical connection ability of grade XI students in solving story problems of linear equations of three variables in terms of problem solving steps according to Polya. The method used in this research is qualitative research. Student connection data was collected using tests and interviews. Then the students' connection data were analyzed based on three types of connections, namely modeling connections, connections between concepts and procedure connections. The results showed that the modeling connection process occurred in the understand the problem and carry out the plan steps. The connection process between concepts occurs at the device a plane and carry out the plan steps. While the procedural connection in the carry out the plan and look back steps. Students' mathematical connection skills are shown in translating problems into mathematical models, connecting between mathematical concepts and procedures. The connection component that has not been mastered by students is not understanding the concept of simplifying equations by grouping similar terms. Students tend to use the operation of moving segments in simplifying the equation.

Keywords: mathematical connection; Polya; problem-solving

INTRODUCTION

Mathematical connections are one of the fundamental skills in mathematics learning. Hatisaru (2023) stated that mathematical connections involve activities carried out by students in linking two or more mathematical concepts. When students can connect mathematical ideas, their understanding becomes deeper and longer-lasting (NCTM, 2000). Mathematical connections can help students recall and understand mathematical concepts and procedures when solving problems (Hendriana et al., 2014). Without mathematical connections, students rely solely on memory for separate mathematical concepts and procedures (Suominen, 2015). This results in shallow understanding and difficulty solving problems involving inter-concept relationships.

Several studies on mathematical connections indicate that the ability to connect mathematics in solving problems is still low. Research by Sari et al. (2018) revealed that students still face difficulties in modeling in the form of quadratic equations. This is supported by Fina et al. (2020), who found that students struggle to create symbolic models. Preliminary studies conducted at one of the public senior high schools (SMA) in Malang Indonesia revealed that students find it challenging to solve word problems involving systems of linear equations with three variables (SPLTV), which require connections between mathematical concepts. Students also tend to have difficulty constructing mathematical models because they do not understand the information provided in the

problems, leading to errors in problem-solving. Hendriana et al. (2014) stated that mathematical connections can help students construct mathematical models and see relationships between concepts used in problem-solving. Students who do not understand relationships between mathematical concepts struggle to solve problems (Anwar et al., 2019). Therefore, mathematical connection skills are essential for students to solve problems involving inter-concept relationships or real-world problems.

Mathematical connection skills can be developed through mathematical problems. One such problem is word problems, which are questions written in sentences describing daily life situations (Johar et al., 2018). However, word problems do not always qualify as mathematical problems. This depends on the individual encountering the word problems. Vula et al. (2017) mentioned that many students find it difficult to solve word problems because the solutions process is complicated. Pandiangan et al. (2021) stated that solving word problems involves two critical processes: problem representations and problem executions. Problem representations involve transforming problems into mathematical models, while problem executions involve mathematical calculations.

Mathematical connections play a crucial role in the problem-solving process. Pambudi et al. (2018) stated that mathematical connections are tools for solving mathematical problems. To solve mathematical problems, students must identify relationships between concepts or theorems used in problem-solving (Romli, 2016). Without mathematical connections, students will find it difficult to solve mathematical problems. Therefore, students must develop mathematical connection skills to solve problems involving conceptual relationships or real-world issues.

The process of mathematical connections occurs in students' minds (Pambudi et al., 2018), making it challenging to trace students' mathematical connection processes directly. Garcia-Garcia and Dodores-Flores (2018) stated that the results of students' mathematical connections can be observed through their work and further investigated through interviews. This study employs Polya's problem-solving steps to trace students' mathematical connection processes, including understanding the problem, devising a plan, carrying out the plan, and looking back. Polya's steps are used in this study because they provide a well-structured framework that avoids overlap in problem-solving steps, making it easier for teachers to understand students' mathematical connection processes.

Etvís (in Busikan, 2008) mentioned five connection processes in problem-solving: modeling connections, procedural-conceptual connections, inter-concept connections, structural connections, and representational connections. Sari et al. (2018) noted that solving word problems involves modeling and calculation. As emphasized by Pandiangan et al. (2021) solving word problems involves two processes: problem representations and problem executions. Based on the above views, this research focuses on the types of connections: Conceptual connections involve linking mathematical concepts to solve problems, procedural connections involve using procedures or algorithms to solve mathematical problems, and modeling connections involve translating situations into mathematical models.

RESEARCH METHODS

The research was conducted at a public senior high school (SMA) in Malang, Indonesia, during the 2023/2024 academic year. This study is descriptive research using a qualitative case study. The material used in this research is the system of linear equations in three variables (SPLTV) related to real-life problems. The subjects of the study are students whose worksheets indicate the presence of mathematical connection processes in each step of Polya's problem-solving stages. The study focused on two participants (then, called as S1 and S2) who were purposively selected based upon the complexity and variation in their mathematical connection processes while solving problems. The purpose of the selection was to gain insight on the participants' relationships to problem-solving via conceptual, procedural, and modeling connections throughout problem-solving stages. S1 and S2 were chosen due to their divergence in symbolic representation, procedural and conceptual mathematical connections, and mathematical writing. The selection of these two participants allowed for multiple contrasts in the study and for the case analysis to gain a deeper understanding of the findings from a qualitative perspective.

The instruments used include a mathematical connection test and an interview guide. The mathematical connection test consists of only one question designed to involve mathematical ideas previously learned. The interview guide is used to assist the researcher during the interviews. Data collection was carried out through tests and interviews. The test aims to determine the mathematical connection processes made by students when solving word problems. The interviews are used to gather additional information about the connections made by the subjects. The connection processes are analyzed based on three types of connections: modeling connections, conceptual connections, and procedural connections. Data analysis in this study refers to the connection type indicators. The indicators of each connection type be shown in Table 1.

Type of Connection	Indikator Koneksi Matematis
Modeling connections	<ul style="list-style-type: none"> • Can express reel objects in the from of the mathematical modeles/mathematics symbols • Can express open sentences in the from of equation
Conceptual connections	<ul style="list-style-type: none"> • Can relate to the concept of three variable linear equations, elimination, substitution method procedures • Can relate the price of shoes per runit to the concept of tax • Can relate package prices to the concept of discounts
Procedural connections	<ul style="list-style-type: none"> • Can apply elimination and substitation methode procedurs • Perform addition, substraction, mulplication,and division in the algebraic from

RESULTS AND DISCUSSION

Based on the research data, the results of the students' tests will be presented, discussing the mathematical connection processes established by the research subjects. Below are the works of S1 and S2.

Understanding the Problem

At the *understanding the problem* stage, S1 demonstrated the ability to identify relevant information from the problem and reorganize it into mathematical representations. In the written response, S1 represented the known information using symbols, namely *P* for *Pantofel*, *O* for *Olahraga*, and *Ob* for *Outbound*, and then formulated the situation into a system of linear equations in three variables. This indicates that S1 was able to establish a modeling connection, namely transforming contextual information into formal mathematical expressions. In addition, S1 also demonstrated an initial conceptual connection by recognizing that the problem structure corresponded to a system of linear equations in three variables (SPLTV). Figure 1 presents S1's written response in the *understanding the problem* stage.

Diket: Pak Andi ingin membeli 3 pantofel, 3 or, 1 outbound.

$$\begin{aligned} \text{Pkt 1} &= 1P + 2O + 1Ob = 265.000 \\ \text{Pkt 2} &= 1P + 1O + 2Ob = 235.000 \\ \text{Pkt 3} &= 2P + 1O + 1Ob = 230.000 \end{aligned} \quad \left[\begin{array}{l} P = \text{Pantofel} \\ O = \text{Olahraga} \\ Ob = \text{Outbound} \end{array} \right]$$

: Setiap paket diskon 5% & tdk pajak
: Pajak 10% peritem

Ditanya: a) Bantu pak Andi mendapat harga paling murah
b) Cara memastikan pembelian sepatu pak Andi sudah benar

Figure 1: Written Response of S1 in the "Understand the Problem" Step

The interview data further confirmed the cognitive processes underlying S1's written work. During the interview, S1 was able to clearly explain all known information in the problem, including the composition of shoe packages, package prices, discount policies, and tax conditions. S1 also accurately identified the goal of the problem, namely determining the cheapest purchasing strategy and verifying whether Pak Andi's purchase decision was correct. The following excerpt illustrates S1's reasoning process:

- P** : What information is known from the problem?
S1 : Mr. Andi wants to buy three pairs of loafers, three pairs of sports shoes, and one pair of outbound shoes. There are also package prices from Matahari store, namely Package 1 costs Rp265,000, Package 2 costs Rp235,000, and Package 3 costs Rp230,000. Every package purchase receives a 5% discount and is not subject to tax, whereas individual item purchases are subject to a 10% tax.
- P** : What is being asked in the problem?
S1 : To help Mr. Andi obtain the cheapest price and determine whether his shoe purchase decision is correct.
- P** : Why did you use *P*, *O*, and *Ob*?
S1 : To form the equations so that it would be easier for me to calculate the shoe prices.
- P** : What kind of equation did you obtain?
S1 : A system of linear equations in three variables.

The interview findings indicate that S1 not only understood the surface information of the problem, but also recognized the mathematical structure embedded within the contextual situation. According to Polya's problem-solving framework, understanding the problem involves identifying known information, determining what is being asked, and recognizing relationships among problem elements. S1 fulfilled these indicators by explicitly restating the facts, defining variables, and constructing mathematical representations. Furthermore, the ability to translate contextual information into symbolic form reflects the

process of mathematical modeling, which is considered an essential aspect of mathematical connection ability. This finding aligns with previous studies suggesting that successful problem solvers tend to construct symbolic representations early in the problem-solving process to support subsequent reasoning and procedural work.

In contrast, S2 demonstrated a different pattern of understanding. Although S2 was able to verbally explain several important facts from the problem during the interview, these ideas were not represented mathematically in the written response. S2 did not define variables, formulate equations, or explicitly identify the problem as a system of linear equations in three variables. The following interview excerpt illustrates S2's reasoning process:

- P** : What information do you know from the problem?
S2 : The price of Package 1 is Rp265,000, Package 2 is Rp235,000, and Package 3 is Rp230,000. There is a 5% discount for package purchases, and a 10% tax if the items are bought individually. Then, Mr. Andi wants to buy 3 pairs of loafers, 3 pairs of sports shoes, and 1 pair of outbound shoes.
P : What is being asked in the problem?
S2 : Mr. Andi wants to buy 3 pairs of loafers, 3 pairs of sports shoes, and 1 pair of outbound shoes at the cheapest price and make sure that his purchase decision is correct.
P : Why did you not write down the known and asked information from the problem?
S2 : I just worked on it directly.
P : Do you often solve word problems without defining variables first?
S2 : Yes.

The interview data suggest that S2 understood the contextual information at a surface level but did not engage in a deeper mathematical structuring process. S2 tended to move directly into procedural work without first organizing the information into mathematical representations. This indicates the absence of a modeling connection at the initial stage of problem solving. From the perspective of Polya's framework, S2 did not fully complete the *understanding the problem* phase because the known and unknown elements were not explicitly represented. Moreover, the lack of variable definition and symbolic representation suggests weak conceptual connections between contextual information and algebraic structures.

This finding is consistent with studies in mathematical connection theory showing that some students can verbally restate information from contextual problems yet experience difficulty transforming that information into formal mathematical models (Jailani et al., 2020; Wijaya et al., 2014). Such students often rely on intuitive or procedural approaches rather than conceptual analysis. Consequently, their problem-solving processes become less structured and more vulnerable to errors in subsequent stages. Thus, the contrast between S1 and S2 demonstrates that the quality of understanding at the initial stage strongly influences the development of mathematical connections and the effectiveness of later problem-solving processes.

Devising a Plan

At the *devising a plan* stage, both S1 and S2 demonstrated the ability to formulate strategies for solving the problem based on the information identified in the previous stage. Both participants planned to determine the price of each shoe item before calculating the tax and discount values. Their strategies involved applying the concepts of systems of linear

equations in three variables through the elimination and substitution methods. This indicates that both participants were able to establish **conceptual connections** by linking algebraic concepts with appropriate mathematical procedures.

S1 demonstrated a systematic strategy for solving the problem. Based on the interview, S1 explained that the first step was to determine the price of each shoe item using elimination and substitution methods. After obtaining the unit prices, S1 planned to calculate the shoe prices after tax and then determine the package prices after discounts before deciding the cheapest purchasing option. The following interview excerpt illustrates S1's reasoning process:

- P** : What concepts did you use to solve this problem?
S1 : Elimination method, substitution method, the concept of calculating discounts, and the concept of calculating taxes.
P : Can you explain how you determined the shoe prices after tax and the package prices after discount?
S1 : The unit shoe price is multiplied by the applicable tax rate. Then, the result is added to the original unit shoe price. Meanwhile, to determine the discounted package price, the package price is multiplied by the discount rate, and then the result is subtracted from the original package price.
P : Then, what strategy did you use to solve this problem?
S1 : First, I determined the price of each shoe using the elimination and substitution methods. After obtaining the prices, I calculated the shoe prices after tax by adding the tax amount to the original price. Then, I determined the package prices after discount by subtracting the discount amount from the original package price. Finally, I determined the cheapest shoe purchase option.

The interview findings indicate that S1 was able to coordinate several mathematical concepts simultaneously, including systems of linear equations, elimination and substitution procedures, percentages, taxes, and discounts. From the perspective of mathematical connection theory, S1 demonstrated conceptual and procedural connections by integrating algebraic procedures with contextual financial situations. According to Polya's problem-solving framework, the *devising a plan* stage involves selecting relevant concepts and organizing them into coherent procedures before executing calculations. S1 fulfilled these indicators by constructing a structured sequence of solution strategies and explicitly explaining the relationships among mathematical concepts involved in the problem (Tay & Toh, 2023).

Similarly, S2 also demonstrated the ability to formulate a solution plan. S2 explained that the problem required several mathematical concepts, namely elimination and substitution methods, tax calculations, and discount calculations. However, unlike S1, S2's explanation was more concise and procedural in nature. The following interview excerpt illustrates S2's planning process:

- P** : What concepts are needed to solve this problem?
S2 : Elimination and substitution methods, the concept of calculating taxes, and discounts.
P : Can you explain how you determined the shoe prices after tax and the package prices after discount?
S2 : The shoe price is multiplied by the applicable tax rate. Then, the result is added to the original unit shoe price. Meanwhile, to determine the discount price, the

package price is multiplied by the discount rate, and then the result is subtracted from the original package price.

P : What strategy did you use to solve this problem?

S2 : First, I determined the price of each shoe using the elimination and substitution formulas. Then, I calculated the shoe prices after tax, followed by the package prices after discount. Finally, I determined the cheapest shoe purchase option.

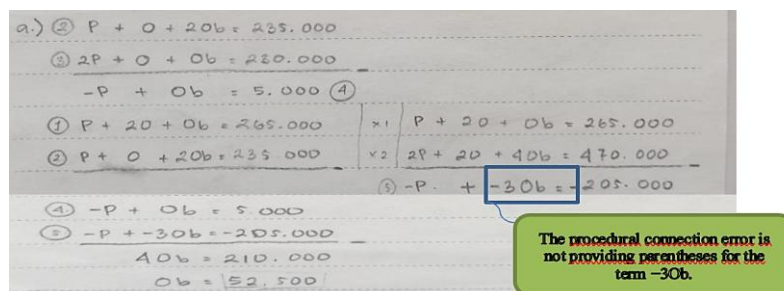
The interview data suggest that S2 was also able to establish conceptual connections between systems of linear equations and elimination–substitution procedures, as well as connect the concepts of percentages, taxes, and discounts with the contextual situation in the problem. Nevertheless, compared to S1, S2’s planning process appeared less elaborated and relied more heavily on procedural descriptions without extensive explanation of the mathematical relationships involved.

Overall, both participants demonstrated the ability to connect multiple mathematical concepts during the *devising a plan* stage. They connected systems of linear equations with elimination and substitution procedures and linked percentage concepts with taxation and discount contexts. These findings support previous studies suggesting that successful mathematical problem solving requires the integration of conceptual understanding and procedural knowledge across different mathematical domains (León del Carmen et al., 2024; Rittle-Johnson & Schneider, 2014; Schneider et al., 2011). However, the quality of the connections differed between participants. S1 exhibited stronger integration between conceptual reasoning and contextual interpretation, whereas S2 relied more predominantly on procedural explanations.

Carrying Out the Plan

At the *carrying out the plan* stage, both S1 and S2 implemented the strategies that had been formulated in the previous stage. The participants performed algebraic operations systematically to determine the unit price of each type of shoe, calculate the prices after taxes, determine package prices after discounts, and finally identify the cheapest purchasing strategy. This stage reflects the emergence of both conceptual and procedural connections, as the participants linked systems of linear equations, elimination and substitution procedures, percentage operations, taxation concepts, discount calculations, and contextual decision-making.

S1 began by determining the price of one pair of outbound shoes using elimination procedures within the system of linear equations in three variables, see Figure 2. S1 eliminated variable O from equations (2) and (3), producing the equation $-P + Ob = 5,000$. S1 then eliminated variable O again from equations (1) and (2), resulting in the equation $-P + -3Ob = -205,000$.



a.)

$$\begin{array}{l} \textcircled{2} P + O + 20b = 235.000 \\ \textcircled{3} 2P + O + Ob = 220.000 \\ \hline -P + Ob = 5.000 \textcircled{4} \\ \textcircled{1} P + 20 + Ob = 265.000 \quad \times 1 \quad P + 20 + Ob = 265.000 \\ \textcircled{2} P + O + 20b = 235.000 \quad \times 2 \quad 2P + 20 + 40b = 470.000 \\ \hline \textcircled{5} -P + -30b = -205.000 \end{array}$$

$\textcircled{4} -P + Ob = 5.000$
 $\textcircled{5} -P + -30b = -205.000$
 $\hline 40b = 210.000$
 $Ob = 52.500$

The procedural connection error is not providing parentheses for the term $-3Ob$.

Figure 2: Written Response of S1 in the "Carrying Out the Plan"

Although S1 omitted parentheses in the expression $-30b$, the interview data revealed that the student understood the intended meaning of the expression. The following interview excerpt illustrates S1's reasoning:

- P** : Please explain the meaning of the equation $-P + -30b$.
S1 : It means $-P$ added to $-30b$.
P : Why did you not use parentheses in the term $-30b$?
S1 : I forgot.

S1 then eliminated variable P from the resulting two-variable equations and obtained $Ob = 52,500$. During the interview, S1 was able to explain the elimination process coherently and interpret the mathematical result contextually:

- P** : What does $Ob = 52,500$ mean?
S1 : The price of one pair of outbound shoes is 52,500.

These findings indicate that S1 successfully connected symbolic manipulation with contextual interpretation. According to mathematical connection theory, this reflects conceptual connections between algebraic procedures and real-world meanings. Furthermore, S1 demonstrated procedural fluency by strategically selecting equations with equal coefficients to simplify the elimination process.

S2 also demonstrated the ability to connect systems of linear equations with elimination procedures to determine the price of outbound shoes, see Figure 3. S2 represented the contextual problem symbolically by defining dress shoes as x , sports shoes as y , and outbound shoes as z . S2 then eliminated variable x from two equations and obtained the equations $y - z = 30,000$ and $y + 3z = 240,000$. By eliminating variable y , S2 obtained $z = 52,500$.

Figure 3: Written Response of S2 in the "Carrying Out the Plan" Step to Determine the Price of Outbound Shoes

The interview excerpt below illustrates S2's reasoning:

- P** : How did you obtain the equations $x + 2y + z = 265,000$, $x + y + 2z = 235,000$, and $2x + y + z = 230,000$?
S2 : By assigning dress shoes as x , sports shoes as y , and outbound shoes as z .
P : What does $z = 52,500$ mean?
S2 : The price of one pair of outbound shoes is 52,500.

Although S2 successfully performed elimination procedures, the interview also revealed partial conceptual limitations. When asked about the type of equations used, S2 stated "I forgot."

This finding suggests that S2 was procedurally capable of performing elimination operations but demonstrated weaker conceptual recall regarding the formal mathematical structure being used. Such findings align with studies showing that some students can execute algebraic procedures correctly without fully understanding the associated formal concepts.

After determining the outbound shoe price, S1 continued by substituting $Ob = 52,500$ into the original equations to determine the prices of dress shoes and sports shoes, see Figure 4. S1 grouped like terms, applied transposition procedures, and connected systems of linear equations in two variables with elimination and substitution methods. S1 obtained $P = 47,500$ and $O = 82,500$.

Figure 4 shows handwritten mathematical work. It starts with three equations:

$$1 = P + 2O + 52.500 = 265.000 \Rightarrow P + 2O = 212.500 \quad (6)$$

$$2 = P + O + 105.000 = 235.000 \Rightarrow P + O = 130.000 \quad (7)$$

$$3 = 2P + O + 52.500 = 230.000 \Rightarrow 2P + O = 177.500 \quad (8)$$

Then, equation (7) is used for substitution:

$$\text{Substitusi: } 47.500 + O = 130.000$$

From this, $O = 82.500$ is derived. This value is substituted into equation (8):

$$2P + 82.500 = 177.500$$

$$-P = -47.500$$

$$P = 47.500$$

Figure 4: Written Response of S1 in the "Carrying Out the Plan" Step to determine the Price of Dress and Sports Shoes

During the interview, S1 explained the reasoning process step by step and interpreted the solutions contextually:

- P** : What do $P = 47,500$ and $O = 82,500$ mean?
S1 : $P = 47,500$ means the price of one pair of dress shoes, and $O = 82,500$ means the price of one pair of sports shoes.

Similarly, S2 substituted $z = 52,500$ into the equation $y - z = 30,000$, resulting in $y = 82,500$. S2 then substituted $y = 82,500$ and $z = 52,500$ into the equation $x + 2y + z = 265,000$ and obtained $x = 47,500$, see Figure 5.

Figure 5 shows handwritten mathematical work. It starts with substitution into the first equation:

$$\text{sub } z \text{ ke } (6)$$

$$y - 52.500 = 30.000$$

$$y = 82.500$$

Then, substitution into the second equation:

$$\text{sub } y \text{ ke } (8)$$

$$x + 2 \cdot (82.500) + 52.500 = 265.000$$

$$x = 265.000 - 217.500$$

$$x = 47.500$$

Figure 5: Written Response of S2 in the "Carrying Out the Plan" Step to Determine the Price of Dress and Sports Shoes

The following interview excerpt illustrates S2's reasoning process:

- P** : Please explain how you solved this problem.
S2 : I substituted $z = 52,500$ into the equation $y - z = 30,000$, then $-52,500$ was moved to the other side and became positive $52,500$. After adding $30,000$ and $52,500$, I obtained $y = 82,500$.
P : What does $y = 82,500$ mean?
S2 : The price of one pair of sports shoes is $82,500$.

These findings indicate that both participants were able to establish procedural connections between substitution procedures and algebraic simplification processes. However, S1's explanations were more systematic and conceptually integrated, whereas S2's explanations were more procedural and operational.

Next, both participants calculated the shoe prices after taxes by connecting the unit prices with the percentage concept of taxation. S1 explicitly showed the complete calculation process by multiplying each unit price by 10% and then adding the result to the original price. During the interview, S1 explained:

- P** : How did you determine the price of each shoe after tax?
S1 : I multiplied 10% by the shoe price, then added the result to the original shoe price.

S1 also explained that values such as 4,750, 8,250, and 5,250 were obtained from multiplying the original shoe prices by 10%. This demonstrates conceptual connections between algebraic results and percentage operations.

In contrast, S2 only presented the final results in the written response without showing complete procedures. However, during the interview, S2 was able to explain the calculation process correctly:

- P** : Please explain how you solved this.
S2 : Each original shoe price was multiplied by 10%, then the result was added to the original shoe price to obtain the shoe price after tax.
P : Why did you not write the complete process?
S2 : I wrote it on scratch paper and did not have time to copy it into the answer sheet because time was almost over.

This finding indicates that S2 possessed procedural understanding but showed limited written mathematical communication compared to S1.

Both participants then calculated the package prices after discounts by connecting package prices with percentage discount concepts. S1 demonstrated the complete written procedure (Figure 6) and explained the process during the interview.

Handwritten calculations for three packages with 5% discounts:

Paket 1 = 265.000 - diskon 5%	
= 265.000 - 13.250	diskon = 265.000 × 5%
= 251.750	= 13.250
Paket 2 = 235.000 - 5%	
= 230.000 - 11.750	
= 218.250	
Paket 3 = 230.000 - 5%	
= 230.000 - 11.500	
= 218.500	

Figure 6: Written Response of S1 in the "Carrying Out the Plan" Step to determine the Price of the Package After the Discount

- P** : Explain the steps for determining the package price after discount.
S1 : The original package price is multiplied by 5%, then the result is subtracted from the original package price.

Similarly, S2 explained:

- P** : Explain how you determined the package price after discount.

S2 : The original package price was multiplied by 5%, then the result was subtracted from the original package price to obtain the discounted package price.

However, S2 again showed only the final results in the written response, while S1 documented the entire procedure systematically. This suggests that S2's procedural understanding was stronger in oral explanation than in written mathematical representation.

Finally, S1 connected all obtained information to determine the cheapest purchasing strategy. S1 reasoned that sports shoes were more expensive when purchased individually due to tax additions and therefore should be prioritized within discounted packages. S1 eliminated Package 2 because it contained two outbound shoes, which did not match the problem requirements. The following interview excerpt illustrates S1's contextual reasoning:

P : Why did you choose Package 1 and purchase two pairs of dress shoes and one pair of sports shoes separately?

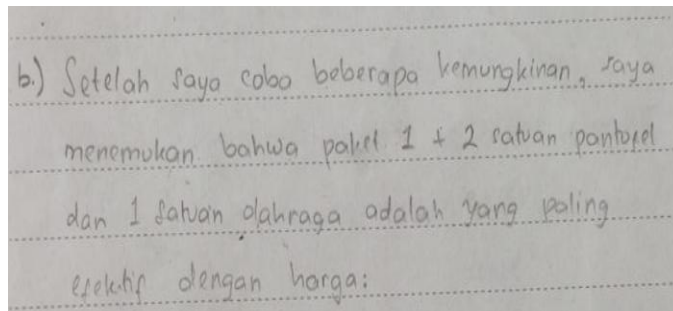
S1 : I first eliminated Package 2 because Mr. Andi only needed one pair of outbound shoes, while Package 2 contained two pairs. Then I compared which shoe was more expensive individually. Sports shoes were more expensive because of taxes, so it was more economical if sports shoes were included in the package.

This finding demonstrates advanced mathematical connections, where S1 integrated algebraic results, taxation concepts, discounts, and contextual constraints into a coherent decision-making strategy. Such reasoning reflects meaningful mathematical understanding because mathematical procedures were not treated as isolated operations but were connected to contextual interpretation and optimization decisions (Lamata et al., 2021; Peled & Zaslavsky, 2008). Overall, the findings at the *carrying out the plan* stage indicate that both participants demonstrated procedural and conceptual connections while solving the problem. However, the quality of these connections differed substantially. S1 showed stronger integration between symbolic representation, conceptual understanding, procedural reasoning, and contextual interpretation. In contrast, S2 demonstrated adequate procedural competence but weaker conceptual articulation and less complete written mathematical communication. These findings support previous research suggesting that successful mathematical problem solving depends not only on procedural fluency but also on the ability to connect concepts, representations, and contextual meanings throughout the solution process (Hatisaru, 2024).

Reviewing (Look Back)

At the *look back* stage, both S1 and S2 rechecked their solutions by comparing several possible purchasing combinations and ensuring that the selected option satisfied the problem requirements. This stage reflects the emergence of **procedural connections**, particularly through the use of addition operations and comparison strategies to verify the correctness and efficiency of the final solution.

S1 reviewed the solution by calculating several alternative purchasing combinations and selecting the cheapest option. In the written response, see Figure 7, S1 concluded that purchasing Package 1 combined with two pairs of dress shoes and one pair of sports shoes separately was the most economical choice.



b.) Setelah saya coba beberapa kemungkinan, saya menemukan bahwa paket 1 + 2 satuan pantofel dan 1 satuan olahraga adalah yang paling efektif dengan harga:

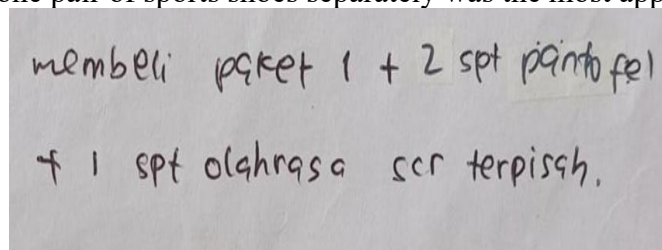
Figure 7: Written Response of S1 in the "Look Back" Step

Although S1 did not present all detailed calculations in the written work, the interview data showed that the student had compared multiple purchasing possibilities before making the final decision.

- P** : Are you sure your answer is correct?
S1 : Yes.
P : How did you make sure that purchasing Package 1, two pairs of dress shoes, and one pair of sports shoes separately was correct?
S1 : I calculated several purchasing options. I found that buying Package 1, then adding one pair of sports shoes separately and two pairs of dress shoes resulted in the cheapest price.
R : Can you mention another purchasing option?
S1 : For example, buying Package 3, then adding one pair of dress shoes and two pairs of sports shoes.

These findings indicate that S1 applied procedural connections by linking contextual facts in the problem with mathematical comparison and addition operations. S1 not only calculated the total prices but also evaluated alternative combinations to verify the validity of the final answer. However, S1 did not completely document all comparison processes in the written response.

Similarly, S2 reviewed the obtained solution by rechecking the purchasing combinations and confirming that the selected option fulfilled the problem requirements. In the written response (Figure 8), S2 stated that purchasing Package 1 combined with two pairs of dress shoes and one pair of sports shoes separately was the most appropriate strategy.



membeli paket 1 + 2 spt pantofel
+ 1 spt olahraga scr terpisah.

Figure 8: Written Response of S2 in the "Look Back" Step

During the interview, S2 explained:

- P** : Is your answer correct?
S2 : Yes.
P : How did you make sure that buying Package 1, two pairs of dress shoes, and one pair of sports shoes separately was correct?
S2 : I calculated all possible purchasing combinations, then selected the lowest price.

The interview findings show that S2 also demonstrated procedural connections by connecting the contextual information in the problem with addition and comparison operations. Similar to S1, S2 verified the final answer by comparing several purchasing possibilities and selecting the most economical option.

Overall, both participants demonstrated the ability to evaluate and verify their solutions during the *look back* stage. However, S1 provided more detailed explanations regarding alternative purchasing combinations, whereas S2 explained the verification process more briefly. These findings suggest that successful problem solving involves not only obtaining a solution but also critically reviewing and validating the reasonableness of the final answer through comparison and contextual evaluation (Rittle-Johnson & Star, 2007; William & Maat, 2020).

CONCLUSION

Based on the results and discussion, referring to the indicators of the mathematical connection process in solving word problems involving systems of linear equations with three variables, the following conclusions can be drawn as follows. The mathematical connection processes that occur when solving word problems involving systems of linear equations with three variables are modeling connections, conceptual connections, and procedural connections. The modeling connection process occurs in the "understand the problem" and "carry out the plan" steps. The modeling connection process happens when the subject assigns symbols to the known information and transforms the problem into a model of a system of linear equations with three variables.

Meanwhile, the conceptual connection process in the "carry out the plan" step occurs when the subject can link the concept of systems of linear equations with three variables to the elimination method, link the concept of systems of linear equations with three variables to the substitution method, and link the concept of systems of linear equations with two variables to the elimination and substitution methods while determining the prices of the shoes before taxes. Additionally, it involves linking the price of each shoe to the concept of taxes to calculate the price after taxes and linking the concept of discounts to the package price to determine the price after the discount.

The procedural connection process occurs in the "carry out the plan" and "look back" steps. The procedural connection process in the "carry out the plan" step occurs when the subject applies operations of addition, subtraction, multiplication, and division in algebraic form and implements the elimination and substitution methods. Procedural connection errors in the "carry out the plan" step are caused by the subject's inaccuracy and lack of understanding of concepts. The component of procedural connection mastered by the subject includes not yet being able to simplify equations by grouping like terms. The subject groups like terms in equations by using transposition operations. Meanwhile, the procedural connection process in the "look back" step occurs when the subject sums all components from each possible shoe purchase to ensure that the shoe purchase strategy is correct and matches the price requested in the problem.

Based on the research findings, the following recommendations are proposed. To foster the development of mathematical connection processes in solving word problems,

teachers are encouraged to emphasize the use of Polya's problem-solving steps during the solution process. By following Polya's steps, teachers can better identify the mathematical connection processes demonstrated by students at each step, making it easier to assess and enhance students' mathematical connection skills. Moreover, teachers should place greater emphasis on the reasoning behind the use of mathematical procedures to prevent students from merely memorizing them as routine rules without understanding their underlying concepts. For future research, it is suggested to further develop this study to uncover or discover new forms of mathematical connections that can emerge during the process of solving word problems.

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