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Students' Intuitiveness in Solving Mathematical Analogy Problems

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ABSTRACT

This study aims to describe students' intuition in solving mathematical analogy problems, which is an essential aspect of analogical reasoning in 21st-century education. The research employed a descriptive qualitative approach. The population in this study was high school students from East Java, with the research subjects comprising students from SMAN 1 Ngoro Mojokerto and SMA Katolik St. Albertus Malang. A total of 31 students participated as samples. Data were collected using an analogy problem related to the system of equations with three variables and a validated interview guideline. The data were analyzed qualitatively by categorizing student responses and identifying intuition characteristics. The results showed that 16 out of 31 students used intuition in solving analogy problems. Their intuition was classified into two main categories: (1) catalytic inference and common sense, shown by 3 students who gave quick and concise answers using prior knowledge, and (2) the power of synthesis and common sense, found in 13 students who gave more elaborate answers based on their understanding and experience. The findings highlight the significant role of intuition in mathematical learning and suggest the need for instructional strategies that support intuitive reasoning.

Keywords: analogical problem-solving; analogical reasoning; intuition

INTRODUCTION

Problem-solving is the ultimate goal of learning mathematics; therefore, every student must possess this ability (Hidayah et al., 2022). According to Utami (2022), problem-solving is a student activity in finding solutions by involving previously acquired knowledge and experience. One form of reasoning involved in problem-solving is analogical reasoning, which is the process of finding solutions by recognizing similarities based on past experiences (Holyoak, 2012). Furthermore, Holyoak (2012) explained that analogical reasoning functions to apply previously learned information to new situations by paying attention to relevant information, extracting relationships, and making mappings across domains to generate conclusions or derive general principles.

Analogical reasoning skills are essential in 21st-century education, particularly for analyzing arguments, drawing conclusions, interpreting information, and understanding concepts (Rohaeti et al., 2019). Holyoak and Thagard (1989) define analogical reasoning as the process of making inferences that one object or situation is similar to another. Sternberg (1977) outlines four stages in analogical reasoning: encoding (recognizing keywords or relevant elements in a problem), inferring (identifying relationships between those elements), mapping (aligning relationships between source and target problems), and applying (using the established analogy to solve the new problem). English (2004) similarly explains these stages, emphasizing the transfer of relationships from familiar (source) to new (target) problems. These stages are reflected in the process of solving analogical problems.

Several studies have used analogical problems that exhibit similarity between the source and target problems (English, 2004; Kristayulita et al., 2018). Utami (2022) describes analogy problems as problems that require students to connect two different things based on their similarities and to draw conclusions. Vendetti et al. (2015) state that a target problem is one that evolves from a source problem and remains related to it. English (2004) adds that the source can be either a written problem or prior problem-solving knowledge.

In mathematics, problem-solving can occur with or without a formal pattern or strategy (Muniri, 2013). Solving problems without a pattern is known as intuitive problem-solving. According to Summers (as cited in Henden, 2004), intuition may arise spontaneously and instantly in one’s consciousness. The knowledge formed from this process is called intuitive knowledge or understanding (Muniri, 2013). Fischbein's research (as cited in Henden, 2004) shows that intuition plays an important role in mathematical problem-solving, as it can happen suddenly, immediately, and often holistically (Muniri, 2013).

The nature of intuitive thinking is categorized into three characteristics by August Mario Bunge (as cited in Henden, 2004). The first is catalytic inference, which refers to the ability to make sudden leaps to conclusions without passing through conscious intermediate steps or visible logical reasoning. This type of intuition allows individuals to arrive at answers quickly, often without being able to explain the reasoning behind them. The second characteristic is the power of synthesis, which involves the ability to combine diverse, even seemingly unrelated, elements into a coherent and meaningful whole. This synthesis requires a well-developed internal logic and mental organization to perceive underlying connections. The third characteristic is common sense, which draws on prior knowledge and everyday experience. It enables individuals to make judgments or decisions based on practical understanding gained through life rather than formal analysis. Together, these three characteristics illustrate the multifaceted nature of intuitive thinking in problem-solving contexts.

Research by Wardhani (2016) found that students with moderate reasoning abilities faced obstacles in the mapping process, while those with lower abilities struggled to understand the problems. Gumanti et al. (2022) also found that students experienced difficulties in choosing appropriate strategies or paths when solving problems. According to Muniri (2018), intuitive thinking acts as a gateway for understanding mathematical concepts and serves as a bridge between existing knowledge and the solution. Thus, intuition-based strategies are needed to connect prior knowledge with problem-solving demands. The stages of analogical reasoning as proposed by Sternberg (1977), with descriptions adapted from English (2004), include: encoding, which involves recognizing each critical piece of information from the problem; inferring, which refers to identifying the relationship between key pieces of information in the target problem; and mapping, which is the process of finding similar relationships between the target problem and the source (prior knowledge), and then drawing inferences based on those similarities.

RESEARCH METHODS

This type of research is descriptive, aiming to describe students' intuitiveness in solving analogy problems. Students' analogy problem-solving was analyzed and described using four stages of analogy reasoning: encoding, inferring, mapping, and applying. This research uses a qualitative approach to investigate and understand the behavior of individuals or groups toward a phenomenon or problem that will be generalized.

The stages in the preparation of instruments in this study begin with compiling the target problem, followed by validating the test questions to expert validators. If the question is valid, then the question can be used for testing, while if the test question is invalid, then there is a need for revision until the question is declared valid and suitable for testing so that then the instrument can be used. The interview guideline contains a list of questions to explore more details or clarify students' intuitions in solving analogy problems. The interview guidelines used in this study also underwent a validation process by experts. The interview guide can be used when it has been declared valid by experts.

The question instrument is in the form of an analogy problem in the form of a story problem about solving a system of three variable equations. Students are asked to determine the items to be purchased with the unit price obtained from solving the system of three variable equations. Data is obtained from student answers and the results of interviews with students.

The first step to collecting research data starts with subject exploration by giving analogy test questions to students of SMAN 1 Ngoro Mojokerto and St. Albertus Catholic High School Malang. The next step was to select students who used intuitiveness in problem-solving. The selected students are students who can solve analogy problems by using intuitive and communicative.

Analysis in solving analogy problems uses analysis of the stages of solving analogies and intuitive problems. The analysis of analogical problem-solving aims to observe the problem-solving process by using the stages of analogical problem-solving. Furthermore, the analysis of students' intuitions in solving problems aims to observe the process of students when looking for solutions to solutions by looking at the intuitions used. The following table describes the intuitions students use to solve analogy problems in this study.

RESULTS AND DISCUSSION

Based on the answers of 31 students, 15 students used analytic in analogical reasoning with the stages of completion starting from encoding, inferring, mapping, and applying. Furthermore, 16 students used intuitively with incoherent stages of analogical reasoning. Based on 16 students, three students answered suddenly with short answers, needed to be more detailed, and used the knowledge they already had. Intuitive with the characteristics of answering suddenly with short answers and not in detail is a catalytic inference character introduced by August Mario Bunge (Henden, 2004). Intuitive, with the characteristics of using steps and rules based on knowledge and experience, is the character of common sense, according to August Mario Bunge (Henden, 2004). Furthermore, 13 other students have the characteristics to answer questions directly, immediately, or suddenly by using the ability to combine formulas based on the knowledge they already have. Intuitive, with the attributes of answering questions directly, immediately, or suddenly by utilizing the ability to combine formulas and algorithms, is a characteristic of the power of synthesis by August Mario Bunge (Henden, 2004). Two categories are obtained based on the solutions made by students who use them intuitively. The first category is students with intuitive catalytic inference and common-sense characters. The second category is students with the power of synthesis and common-sense characters. Based on these two categories, one student was chosen as a representative to be analyzed. The following research informants based on the results of the answers to analogy problems and interviews can be seen in Table 1.

Table 1. Research Subjects

No	Intuitive Category	Number of Students	Representative	Initials
1	Catalytic Inference and Common Sense	3	1	S-1
2	Power of Synthesis and Common Sense	13	1	S-2

S-1

The following explains S-1's process in solving the analogy problem. Image 1 shows that S-1 calculates the price of each bundling package before the discount.

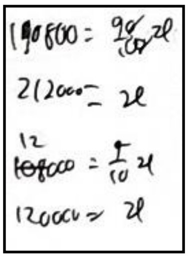

Figure 1. Inferring Stage S-1

When confirmed, S-1 explained that she saw three bundling packages with three different items and immediately connected the problem by finding the unit price using SPLTV. S-1 could connect important information and determine the solution spontaneously. After realizing a ten percent discount, S-1 did Encoding to find important information from the problem. S-1 guessed that the question asked for the price of the unit item before the discount, so he looked for the price of the package before the discount and used SPLTV to solve it. S-1 used x for the price of the pencil case, y for the ballpoint pen, and z for the notebook, then proceeded to the inferring stage.

Furthermore, S-1 also wrote down the three equations that corresponded to the suspected problem as the mapping stage seen in Figure 2.

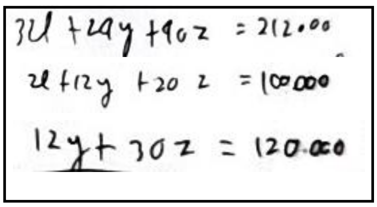

Figure 2. Mapping Stage S-1

S-1 explained, "Package one contains three pencil boxes, twenty-four pens, forty notebooks, so three x plus twenty-four y plus forty z equals two hundred and twelve thousand. Package two contains one pencil box, twelve ballpoint pens, and twenty notebooks, so one x plus twelve y plus twenty z equals one hundred thousand; package three contains twelve ballpoint pens and thirty notebooks, so twelve x plus thirty y equals one hundred and twenty thousand". Based on S-1's explanation, S-1 mapped the solution steps determined at the inferring stage. Next, S-1 calculated the solution to the equation that had been made. S-1 wrote the solution process as follows in Figure 3.

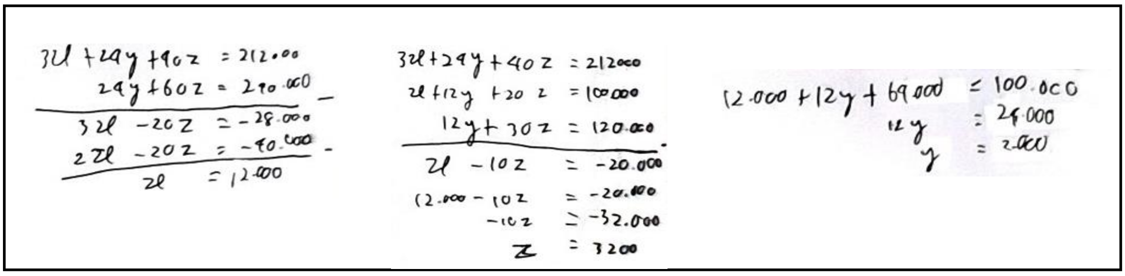

Figure 3. Applying Stage S-1

S-1 explained, "This equation, x twelve y twenty z, is eliminated with the equation twelve y thirty z, the result is x minus ten z equals min twenty thousand. Then this equation (while pointing to the equation in question) three x two four y forty z is eliminated with the equation of bundling three multiplied by two, so it is two four y sixty z, and the result is three x min twenty z equals min twenty-eight thousand. Then three x min twenty z equals min twenty-eight thousand (while pointing to the result). I subtracted this result but multiplied it by two; x minus ten z equals min twenty thousand multiplied by two, so two x minus twenty z equals min forty thousand, and the result is x equals twelve thousand. Then I substitute x in x minus ten z equals min twenty thousand. So twelve thousand minus ten z equals twenty thousand, min ten z equals min thirty-two thousand, z equals three thousand two hundred. Then I substitute x and z in this equation (while pointing to the equation of x+12y+20z=100000), so twelve thousand plus twelve y plus sixty-four thousand equals one hundred thousand. So twelve y equals twenty-four thousand, y equals two thousand". Based on S-1's explanation, S-1 did the applying stage.

After S-1 confirmed at the applying stage that he could find the solution to SPLTV, S-1 explained, "I reread it, and that's why I underlined that it was true that the price of the unit item returned to the normal price. "Based on this explanation, in the middle of applying, S-1 looked back at the problem to confirm the suitability of the solution he had determined for the issue at hand. Furthermore, S-1 wrote, as follows in Figure 4.

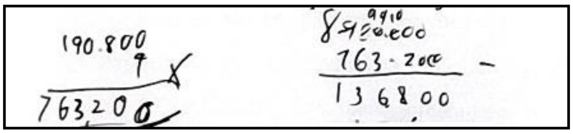

Figure 4. Encoding Stage S-1

The writing is suspected to be Encoding, then confirmed to S-1. S-1 explained, "It turns out that the problem is looking for what items Ms. Eva will buy after buying four bundling packages. So, one hundred ninety thousand eight hundred multiplied by four, the result is seven hundred sixty-three thousand two hundred. Then I subtracted nine hundred thousand from seven hundred sixty-three thousand two hundred, so the remaining one hundred thirty-six thousand eight hundred. Ms. Eva's groceries are also more than five hundred thousand, so the shipping can be ignored". Furthermore, S-1 wrote the following in Figure 5.

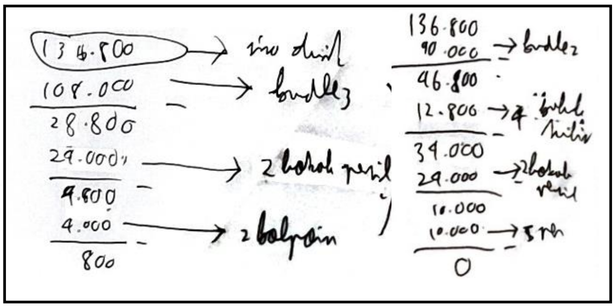

Figure 5. Applying Stage S-1

S-1 explained, "The remaining one hundred thirty-six thousand eight hundred I subtracted with one hundred eight thousand that was bundling three, the remaining two eight hundred I subtracted again with twenty-four thousand that was two pencil boxes, the remaining four thousand eight hundred, I subtracted again with four thousand that was two ballpoint pens, the remaining eight hundred rupiahs. Again, I thought the remainder should be zero rupiah, so I looked for another combination. Finally, I tried one hundred and thirty-six thousand eight hundred. I subtracted with ninety thousand that used bundling two, the remaining forty-six eight hundred, then again with twelve thousand eight hundred that four notebooks, the remaining thirty-four thousand, then I subtracted with twenty-four thousand that two boxes of pencils. Finally, I subtracted with ten thousand, that is five pens, and it turns out right, the rest can be zero".

S-2

The following explains S-2's process for solving the analogy problem. In Image 6, S-2 writes SPLTV. This activity is the mapping stage. After being confirmed, S-2 explains, "I immediately, ma'am, I must use SPLTV with the price before the discount, so I immediately looked for it; the price before the discount was easy, ma'am; it was only ten percent." Based on S-2's explanation, S-2 mapped after determining the solution form to be applied.

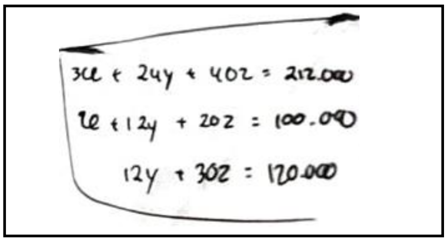

Figure 6. Inferring Stage S-2

S-2 explained, "Package one contains three pencil boxes, twenty-four ballpoint pens, forty notebooks, so three x plus twenty-four y plus forty z equals two hundred and twelve thousand. Package two contains one pencil box, twelve ballpoint pens, and twenty notebooks, so one x plus twelve y plus twenty z equals one hundred thousand; package three contains twelve ballpoint pens and thirty notebooks, so twelve x plus thirty y equals one hundred and twenty thousand". Based on S-2's explanation, S-2 mapped the steps of the predetermined solution. Next, S-2 is calculated to find the solution of the equation. S-2 wrote the solution process as follows in Figure 7.

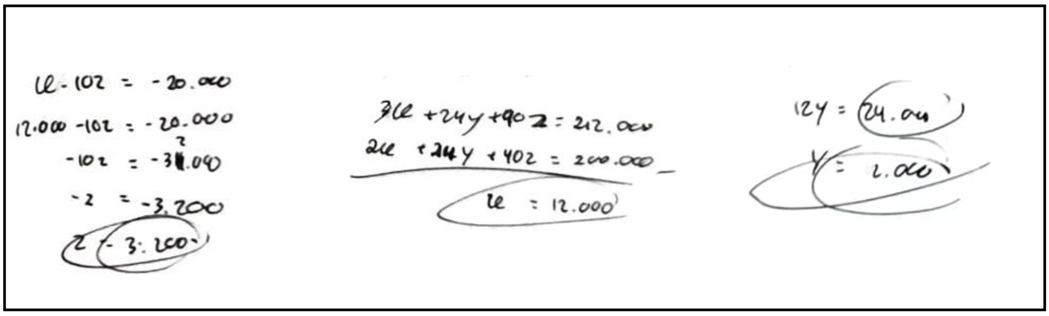

Figure 7. Applying S-1 Stage

S-2 explained, "This equation, three x two twenty-four y forty z, is linearized with the second equation, but I have not multiplied by two, so two x plus two four y plus 40 z equals two hundred thousand; the result is x equals twelve thousand. I put it into the equation x min ten z, the equation I got from subtracting this equation (pointing to the equation x+12y+20z=100000) and this equation (pointing to the equation12y+30z=120000), the result is x minus ten z equals min twenty thousand, so I get z is three thousand two hundred. Then I substituted x and z in this equation (pointing to the equation), which is twelve thousand. x+12y+20z=100000), the result is twelve thousand plus twelve y plus sixty-four thousand equals one hundred thousand. So twelve y equals twenty-four thousand, y equals two thousand". Based on S-2's explanation, S-2 did the applying stage.

After S-2's confirmation at the applying stage to be able to find the solution of SPLTV, S-2 explained that "here there is free shipping ma'am because Ms. Eva bought more than five hundred thousand, Ms. Eva bought four bundles costing seven hundred sixty-three thousand two hundred, the remaining one hundred thirty-six thousand eight hundred. "Based on this explanation, in the middle of applying, S-2 looked back at the problem to find other vital information to continue solving the problem. The reason is S-2's encoding stage, as shown in Figure 8.

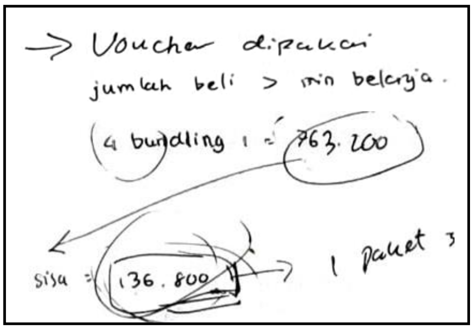

Figure 8. Encoding Stage S-2

Furthermore, S-2 wrote the following in Figure 9.

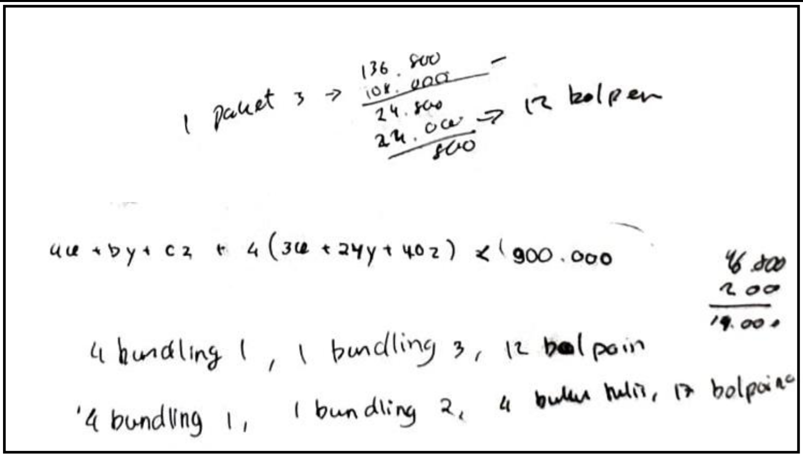

Figure 9. Applying S-1 Stage

S-2 explained, "The remaining one hundred and thirty-six thousand eight hundred I subtracted with one hundred and eight thousand that is bundling three, the remaining two four eight hundred I subtracted again with twenty-four thousand that is twelve pens, the remaining eight hundred rupiahs." Based on the explanation, S-2 continued the applying stage by determining what items Mrs. Eva would buy with less than one thousand rupiah remaining.

The findings of this study reveal how intuitive thinking manifests in students' analogical problem-solving, aligning with theoretical perspectives on intuition in mathematics learning. Students with an intuitive cognitive style tend to understand problems by reading them once, using information they know spontaneously, and applying unstructured problem-solving methods, in contrast to systematic students who follow planned and structured steps (Azizah et al., 2021). According to Henden (2004), intuition is not a step-by-step process but rather a spontaneous cognitive activity, often guided by necessity and rational insight. This helps explain why students in this study demonstrated jumps in reasoning rather than linear progression—indicative of intuitive rather than algorithmic processing.

The process of problem-solving by the subject is not gradual and tends to jump around, in line with the views of Plato and Aristotle (Henden, 2004), that intuition does not occur step by step. The problem-solving process is often not linear or gradual, but can jump between stages as needed, in line with the concept of intuition in philosophical thought such as Plato and Aristotle (Kant & Newell, 1984). The characteristics of the intuitive process in problem solving are marked by the sudden emergence of solutions (insight), where individuals suddenly find the answer after experiencing a pause or incubation period of the problem. This phenomenon does not go through a gradual thinking process, but occurs spontaneously and quickly (Gilhoolya, 2016). In this process, the role of the subconscious is very important because during incubation, the brain continues to work unconsciously to process information and finally produces a solution intuitively (Gilhoolya, 2016).

Intuition plays a vital role in a variety of contexts. In the realm of creativity and innovation, intuition contributes significantly to generating unconventional solutions, making it essential for creative problem solving (Gilhoolya, 2016). In complex and dynamic situations, intuition helps individuals to understand the big picture holistically, enabling faster and more adaptive decision-making (Hallo & Nguyen, 2021). In the context of mathematics and education, intuition has been shown to enhance problem-solving abilities at various levels of education, as it enables students to connect existing knowledge to new problems more flexibly and efficiently (Suwarto et al., 2023). Meanwhile, those demonstrating the power of synthesis exhibited more structured, though still intuitive, problem-solving—a process that aligns with English's (2004) view of analogical reasoning as involving recognition, mapping, and application.

Moreover, the use of prior knowledge to navigate problem spaces confirms the cognitive role of analogical reasoning in mathematics education. This supports research by Holyoak (2012) and Vendetti et al. (2015), which describes analogy as a tool to transfer known information to unfamiliar contexts. The students' success in making such transfers, despite incomplete or inconsistent stages, points to intuition as a bridge between conceptual understanding and procedural execution.

However, the research has several limitations. First, the qualitative nature and limited sample size (16 intuitive thinkers out of 31 students) restrict the generalizability of the results. Second, the absence of a controlled comparison with non-intuitive problem solvers makes it difficult to evaluate the relative effectiveness of intuition over other strategies. Lastly, the categorization of intuition relied heavily on observable behavior during interviews, which might not fully capture students' internal cognitive processes. Further research is needed to investigate how instructional strategies can support the development of productive intuitive thinking, particularly in learners who rely more on procedural or rigid strategies. Exploring how intuition interacts with metacognitive awareness could also offer new insights for designing more effective mathematics education interventions.

CONCLUSION

Based on the research, two subjects (S-1 and S-2) solved the analogy problem by relying on intuition, although using different approaches. S-1 started with inferring and encoding, then mapping, applying, and applying to ensure the solution fits the problem. S-2 immediately did mapping, then applied and Encoded in the middle of applying to find other important information. S-1 used "power of synthesis" and "common sense" intuition, while S-2 used "catalytic" and "common sense ."Both subjects could solve problems with intuition based on prior knowledge. This study confirms that intuition and previous knowledge are essential in solving mathematical problems, although the steps are sometimes gradual.

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