



Students' Specializing Thinking in Solving Arithmetic Sequences and Series Problems

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<i>Submitted: 22-12-2024</i>	<i>Revised: 13-03-2024</i>	<i>Accepted: 05-06-2024</i>	<i>Published: 30-06-2024</i>
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ABSTRACT

Specializing thinking is a mathematical thinking process that is very important in the mathematics learning process. Specializing thinking is thinking by starting with special things. The application of specializing thinking can be an effective strategy for teachers to improve students' mathematical thinking abilities in dealing with various problems. This research aims to describe students' specializing forms of thinking in solving problems of arithmetic sequences and series. This research method is descriptive qualitative research with a case study approach. The subjects in this research were 2 class X high school students who successfully solved arithmetic sequence and series problems. The instruments used are tests and interviews. The data in this research are the results of the subject's work and the results of interview transcripts. The research results show that two forms of specializing thinking were found in solving arithmetic sequences and series problems, that is explicit schematic representation and implicit schematic representation. It is hoped that the results of this research can provide a valuable contribution to mathematics teachers in designing more meaningful learning.

Keywords: arithmetic sequences and series problems; mathematical thinking; specializing

INTRODUCTION

One important aspect of mathematics that students need to have is the ability to develop their thinking skills (Izzatin et al., 2020; Komala & Sarmini, 2020; Tsany et al., 2020). This thinking ability has significance in its application at elementary school, middle school, and university levels (Kurniati & Zayyadi, 2018). Without realizing it, upper secondary students should be able to solve problems correctly where based on Piaget's stages of development, at the age of 12 years and above someone is expected to be able to carry out formal or abstract thinking processes (Kurniati & Zayyadi, 2018; Faizah et al., 2020). To help students understand complex problems, students are directed to think mathematically.

Mathematical thinking is a type of thinking that is based on processes such as learning, analyzing, arguing, finding formulas and solutions, creating new formulas and solutions, and developing new methods for solving mathematical problems (Kenjayeva, 2023; Septian & Rahayu, 2021; Setiawan et al., 2021). In learning mathematics, mathematical thinking can be used as an important aspect and fundamental goal of education (Stacey, 2006). Therefore, it is hoped that these basic goals can be achieved effectively through the application of good mathematical thinking.

According to Mason et al. (2010), mathematical thinking is a dynamic process that can broaden and deepen understanding by increasing the complexity of ideas that can be completed. In mathematical thinking, four fundamental processes are often followed (Stacey, 2006). The basic processes in mathematical thinking include specializing, generalizing, conjecturing, and convincing (Stacey, 2006). Specializing thinking is a mathematical thinking process that takes the form of specializing in the problem at hand. Generalizing thinking is a mathematical thinking process that takes the form of observing patterns. Conjecturing thinking is a mathematical thinking process whose form is to provide temporary conclusions based on observations of patterns or generalizing results. Convincing thinking is a mathematical thinking process that takes the form of an explanation to the reader so that the reader is confident in the results obtained. However, in this research, mathematical thinking which will be explored in more depth only focuses on the specializing thinking process whereas mathematical thinking refers to Stacey's framework.

Research related to specializing thinking was conducted by Putri et al. (2020) which shows that there is a first mathematical thinking process, namely specializing. The results found were that students with high abilities had better specialization. Specializing helps students understand questions by emphasizing students' ability to clarify ideas about a pattern. Apart from that, students who have good specialization by working on problems systematically will easily solve a problem. In line with previous researchers, Iswari et al. (2019) show that there are abstraction activities in mathematical thinking, namely observing patterns, specializing, generalizing, conjecturing, and testing an assumption from the results of the subject's answers. The results found were that specializing emerged from solving problems by looking at certain examples (special things). However, the discussion presented still focuses on subject descriptions which only rely on solving problems through formulas without understanding prior knowledge and previous concepts, thus making the character of each subject less exposed and less interesting. Therefore, the researcher tried to make updates to the research carried out by linking subject specializing thinking with a form of schematic representation.

Previous research on schematic representation was conducted by Anwar et al. (2019) and Anwar & Rahmawati (2018), the results of both studies concluded that the formation of the schematic representation process begins with reading the questions repeatedly, identifying the questions by forming a scheme, and creating schematic images as a structure for one's thinking in solving problems. So it can be concluded that the process through which the schematic representation is formed leads to a specializing thinking process which begins by specializing in certain cases and then continues with forming general cases. This is supported by Stacey (2006) who defines specializing as specialization. In other words, it is possible that in this research the problem solving process created by students will lead to a schematic representation. So, in this research, mathematical thinking will be demonstrated by applying Stacey's framework, namely specializing thinking which focuses on the case of subject comparison through descriptions in the form of schematic representations used by students in solving problems with the concept of arithmetic sequences and series.

Arithmetic sequences and series are one of the advanced materials related to number patterns. Patterns are usually considered as evidence in mathematical thinking (Singer & Voica, 2022). Meanwhile, stimulating students' mathematical thinking processes can be done by applying problem context to numbers and basic algebra (Stacey, 2006). In line with the opinion of Singer & Voica (2022), patterns can be classified into several categories, including number patterns, geometric patterns, repeating patterns, and so on. If by understanding patterns, students can explore, generalize, and represent these patterns in mathematics (Pang & Sunwoo, 2022), then in mathematics learning it is recommended to introduce number patterns so that the development and improvement of children's reasoning abilities are more advanced when faced with a problem (Wilkins et al., 2022).

A problem is an obstacle and difficulty or challenge that requires a solution or resolution of the problem (Abidin, 2017). In mathematics, a problem is a mathematical problem that cannot be solved through routine procedures, this means that the type of problem referred to here is a problem that has high cognitive demands (Suseelan et al., 2023), so solving these mathematical problems requires experimentation using several steps to find a suitable solution. This research aims to identify patterns from different sides of the concept of arithmetic sequences and series. The mathematical content obtained from students is the result of solving arithmetic sequences and series problems in the form of schematic images illustrated directly and indirectly, thus making this research different from previous studies that have discussed mathematical thinking processes.

RESEARCH METHODS

This research is included in qualitative research by applying a case study approach, where a case is analyzed in detail to produce a theory (Creswell, 2012). The case observed is a specializing thinking process which is described in schematic form in different ways, that is representations that guide students' minds to focus on solving problems directly and indirectly.

A total of 7 classes The subjects who will be explored for specializing thinking are selected after observing the characteristics of the data found by the researcher. The criteria for selecting research subjects, namely students who have received sequence and series material, can solve problems to give rise to specializing thinking criteria in the results of working on the problem and can communicate well. After observing based on the characteristics of the data, from the 7 students, 2 students were obtained whose problem solving results were by specializing thinking. Thus, these two students could be selected as subjects in this research through the application of the purposive sampling method. The instruments used in this research were arithmetic sequences and series problems which consisted of two problems (see Figure 1) and an interview guide.

Mrs. Ida is a coal cloth craftsman in Bangka Belitung. The difference in the increase in the number of pieces of coal cloth completed by Mrs. Ida each month is constant. If in the 5th month, the number of coal cloth completed was 18 pieces and in the 9th month it was 30 pieces. Calculate it:

- The difference in the number of pieces of coal cloth completed each month
- Many pieces of coal cloth were finished for the first time

Figure 1. Research Instrument

The data collection techniques applied are test results and interview results. The test results are grouped based on similar answer patterns and analyzed by referring to Stacey's framework regarding specializing thinking pairs by containing several indicators. Indicators of specializing thinking are choosing information that is considered important, making simple pictures, and trying out certain numbers. Furthermore, the method used in the interview was unstructured, which was conducted online with the subject, the aim was to obtain specific answers from the subject (Creswell, 2012) regarding specializing thinking in solving arithmetic sequences and series problems.

In the context of this research, data was analyzed qualitatively. Qualitative data analysis is an effort to manage, organize, and filter data into units that can be organized, synthesize them, and identify patterns that emerge. The hope is that the important points presented can be taken from the results and can be studied by readers. The data analysis technique applied in this research was taken from the theory of Miles & Huberman (1994). The analysis stages are shown in Table 1.

Table 1. Qualitative Data Analysis of Students' Specializing Thinking

Analysis Stages	Description
Data Reduction	Sorting data based on indicators of specializing thinking where the results of thinking are represented in schematic form, either directly or indirectly.
Data Display	The data presented is a description of students' specializing forms of thinking in solving problems of arithmetic sequences and series.
Drawing Conclusions	The conclusion was that there was a form of schematic representation in the results of the subject's work and thoughts.

Based on Table 1, it can be described that the data analysis steps by the Miles & Huberman (1994) framework are as follows:

1. Data Reduction

Data reduction or decomposition aims to determine specializing forms of thinking that are considered relevant to schematic representation. In addition, reduction is needed to filter out information that is not important in this research.

2. Data Display

Data display is carried out in more detail and clarity, the aim is to provide a way that makes it easier to observe the existence of schematic representations in the specializing thinking process.

3. Drawing Conclusions

By referring to the findings of this research and the validity strengthened by previous research, it can be concluded that there is a schematic representation, such as making schematic drawings of work results and ideas that are manifested in designing problem solutions.

RESULTS AND DISCUSSION

The research results show that there are two forms of specializing thinking in solving arithmetic sequences and series problems, namely: (1) explicit schematic representation, and (2) implicit schematic representation. A schematic representation is a representation that

From the interview excerpt, it can be seen that from the problem the first activity carried out by the subject was selecting important information such as seeing something that was known from the questions related to the fifth month there were 18 cual cloths and the ninth month there were 30 cual cloths. Second, the subject begins to look for the difference from the number pattern, that is 3, so that the subject can determine the value of the difference using the pattern $18 + 4n = 30$. Third, the subject tries to find lots of cloth in the first month by drawing the rows like a number line. The way the subject finds lots of cloth is by starting to look for lots of clothes from the eighth month, because in the ninth month the number of pieces of cloth is 30 then in the eighth month $= 30 - 3 = 27$, then continue again by trying the numbers in the seventh month, that is the seventh month $= 27 - 3 = 24$, and so on until the first month where the result is 6. Finally, the subject checked manually by adding 3 every month and it turned out that the results were the same as those depicted in the scheme so the subject was sure that the results obtained were correct. The aim of the subject in drawing the sequence into a scheme is to make it easier for the subject to solve the problem. This process is called explicit schematic representation. So it can be concluded that subject 1 solves arithmetic sequence and series problems by using explicit schematic representations in these arithmetic sequence and series problems. This process can be seen in Figure 3 below.

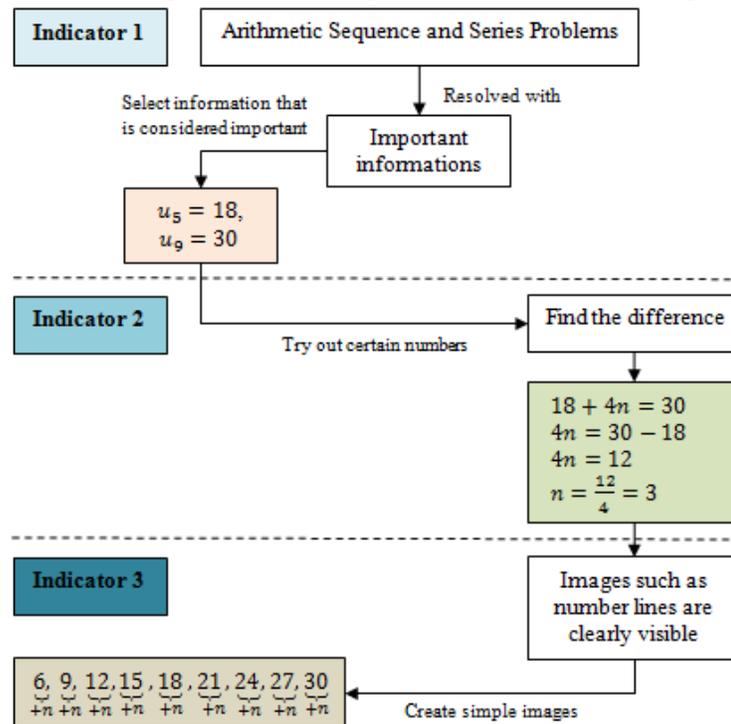


Figure 3. Thinking Specializing Subject 1 through Explicit Schematic Representation

From Figure 3 it can be seen that the subject's specializing thinking in solving arithmetic sequences and series problems through explicit schematic representation fulfills three indicators. The first indicator is selecting information that is considered important. In

this first indicator, the subject focuses on known information such as $u_5 = 18$, $u_9 = 30$ so that it makes it easier for the subject to formulate problem solving according to the understanding that has been obtained previously. The second indicator is trying out certain numbers. In this indicator, the subject begins to try to enter numbers that match the pattern that the subject has found so that the results obtained will form an arithmetic sequence. The third indicator is to create simple images. In this indicator, the subject focuses on describing the results of solving arithmetic sequences and series problems directly by making illustrations such as the shape of a number line.

Implicit Schematic Representation

The first process of solving arithmetic sequences and series problems is through implicit schematic representation. The results of the work of the second subject (S2) in solving arithmetic sequences and series problems with implicit schematic representation can be seen in Figure 4.

① a.) Diketahui : bulan 5 = 18 helai
 bulan 9 = 30 helai
 selisih banyak helai dari bulan 5 - bulan 9 = $30 - 18$
 $= 12$ helai (4 bulan)

Jawab = Banyak helai yg dibuat tiap bulan = $12 : 4$
 $= 3$ helai (tiap bulan)

b) bulan ke 4 = $18 - 3 = 15$ helai
 - " - ke 3 = $15 - 3 = 12$ helai
 - " - ke 2 = $12 - 3 = 9$ helai
 - " - ke 1 = $9 - 3 = 6$ helai (Bulan pertama buat 6 helai)
 untuk pertama kalinya berarti buat 3 helai karena Bulan 1 ia membuat 6 helai

Figure 4. Answers of Subject 2 in Solving Arithmetic Sequence and Series Problems

From Figure 4 on the problem of sequences and arithmetic series, it can be seen that the subject writes down the process of solving the problem by starting to solve the problem from specific things, then proceeding to a general form such as the subject writing down important information known from the question that the fifth month has 18 pieces and the ninth month has 30 pieces. Next, the subject determines the difference by starting to subtract the number of pieces of cloth in the ninth month minus the fifth month, that is $30 - 18 = 12$ pieces and concludes that 12 is the difference for 4 months, then the difference for each month is $\frac{12}{4} = 3$. Then the subject makes a pattern starting from the fourth month, that is $18 - 3 = 15$, continued with the third month, that is $15 - 3 = 12$, the second month that is $12 - 3 = 9$, and until the first month, that is $9 - 3 = 6$. So the number of pieces of cloth in the first month is 6. The way the subject describes the relationships in solving arithmetic sequences and series problems can be known more fully through the following excerpt from the researcher's (P) interview with subject 2 (S2).

P : From the problem given, try to explain how you went about solving the problem in that question?

S2 : It is known from the question that the number of pieces of cloth in the fifth month is 18 pieces and the ninth month is 30 pieces, Next, I looked for the difference by reducing the number of pieces of cloth in the ninth and fifth months, the result was 12 pieces for 4 month, So the difference each month is $\frac{12}{4} = 3$ pieces. Next, I tried looking starting from the fourth month = $18 - 3 = 15$ pieces, the third month = $15 - 3 = 12$ pieces, the second month = $12 - 3 = 9$ pieces, and the first month = $9 - 3 = 6$ pieces. So in the first month there are 6 pieces of cloth.

P : Then why did you start looking from the fourth month?

S2 : Because in the fifth month there were 18 pieces of cloth, so I started from what was known first.

From the interview excerpt, it can be seen that from the problem of sequences and arithmetic series, the first activity carried out by the subject was to select important information such as seeing something that was known from the problem that in the fifth month there were 18 cloth and in the ninth month there were 30 cloth. Second, the subject begins to look for the difference by reducing the number of pieces of cloth in the ninth and fifth months, the result is 12 pieces for 4 months, so the difference for each month is $\frac{12}{4} = 3$ pieces. Third, the subject tried to search starting from the fourth month = $18 - 3 = 15$ pieces, the third month = $15 - 3 = 12$ pieces, the second month = $12 - 3 = 9$ pieces, and the first month = $9 - 3 = 6$ pieces. So in the first month there are 6 pieces of cloth. This process is called implicit schematic representation. So it can be concluded that subject 2 solves arithmetic sequences and series problems using implicit schematic representation. This process can be seen in Figure 5 below.

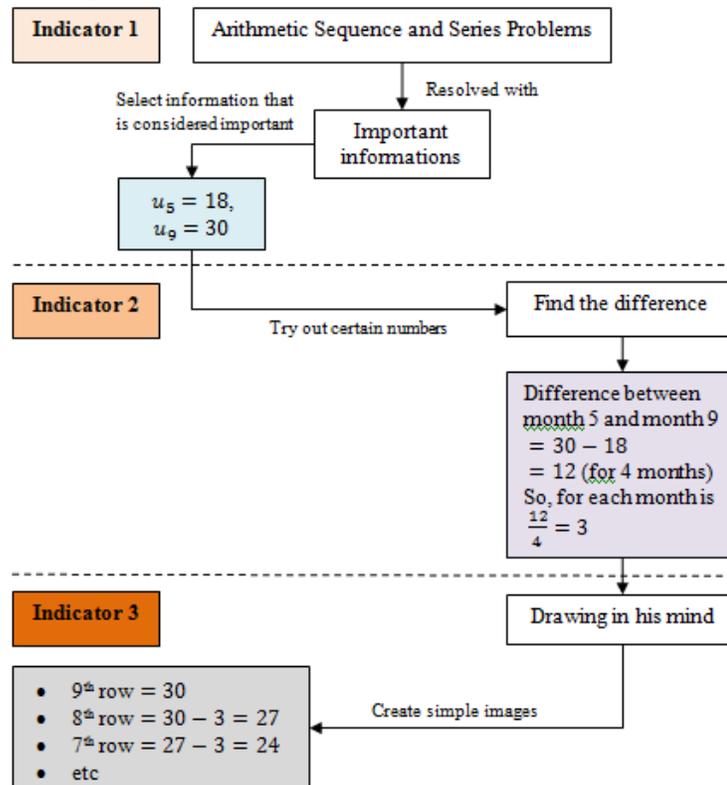


Figure 5. Thinking Specializing Subject 2 through Implicit Schematic Representation

From Figure 5 it can be seen that the subject's specializing thinking in solving problems of sequences and arithmetic series through implicit schematic representation fulfills three indicators. The first indicator is selecting information that is considered important. In this first indicator, the subject focuses on known information such as $u_5 = 18$, $u_9 = 30$ so that it makes it easier for the subject to formulate problem solving according to the understanding that has been obtained previously. The second indicator is trying out certain numbers. In this indicator, the subject begins to try to enter numbers that match the pattern that the subject has found so that the results obtained will form an arithmetic sequence. The third indicator is to create simple images. In this indicator, the focus subject describes the results of solving the arithmetic sequence problem indirectly, which means it only occurs in the subject's mind.

In general, it is found that the process of specializing thinking on this subject has found two forms of thinking in solving problems of arithmetic sequences and series, the first is an explicit schematic representation and the second is an implicit schematic representation. In this research, the form of explicit schematic representation presented is that the focus subject describes the results of solving arithmetic sequences and series problems directly by making illustrations such as a number line. Meanwhile, the form of implicit schematic representation shown is the opposite, in that the focused subject describes the results of solving the problem of arithmetic sequences and series indirectly, which means it only occurs in the subject's mind. The results of this research are in line with the results of previous research which shows that students create schematic representations by making schematic images containing a problem framework and equipped with several key information related to the problem (Anwar, Rahmawati, et al., 2019). Furthermore, students who succeed in forming schematic representations can present schematic information and images more concisely, accurately, and precisely (Anwar et al., 2021; Anwar et al., 2019). As observed in previous research, the results of this study also show that subjects who solve sequence and arithmetic series problems through schematic representation are also able to express the solution concisely through the presentation of schematic images.

The method used by the two subjects by applying previously acquired knowledge through schematic representation is considered quite effective because it can give rise to specializing thinking when solving arithmetic sequences and series problems. Various research results also show that the presence of schematic representations will have a clear positive effect on overall student performance and that most students succeed in reusing the representations they encounter to solve new problems (Fagnant & Vlassis, 2013). This is in line with Estes & Wisniewski who explain that schematic structures are based on concepts and principles that emerge from previous experience, thus involving the construction of relationships that connect different concepts (Kim & Jung, 2023). By creating schemes, students can understand questions and problems well (Anwar & Rahmawati, 2018). Fagnant & Vlassis support this view by stating that schematic representation has a crucial role in solving complex problems because it allows students to describe the essence of the problem in schematic form (Anwar & Rahmawati, 2022). Therefore, through the use of schematic representation, students can extract key information

and understand the relationship between the information contained in the problem (Thevenot & Barrouillet, 2015 dalam Anwar & Rahmawati, 2022).

CONCLUSION

Based on research that has been carried out by exploring specializing thinking in solving arithmetic sequences and series problems, it was found that there are two specializing thinking, namely explicit schematic representation in subject 1 and implicit schematic representation in subject 2. A schematic representation is a representation that describes relationships in explaining problem-related situations. Explicit schematic representation means a representation that describes relationships in solving problems directly. Meanwhile, implicit schematic representation means a representation that describes relationships in solving problems indirectly. By finding a specializing way of thinking through schematic representation, it is hoped that it can contribute to mathematics teachers in designing more meaningful learning. Apart from that, these findings can be a direct reference for teachers in developing learning tools, evaluation tools, and learning media that suit the needs of students' cognitive levels.

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