



Middle School Students' Proportional Reasoning Ability in Solving Proportional and Non-proportional Problem

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ABSTRACT

The objective of this descriptive qualitative study is to describe students' proportional reasoning ability in solving each type of proportional problem, namely missing-value problem, numerical comparison problem, and qualitative comparison problem. A non-proportional problem was also included in this study to assess students' ability in one of the indicators used to measure proportional reasoning ability. The instrument used in this study consists of mathematical word problems on the proportion concept. The participants involved nine eighth-grade students, who were chosen from a total of 49 eighth-grade students at a school in Bandung City. These students were selected using a technique known as purposive sampling, by looking at the answers of students who best represent the answers of other students. The gathered data were analyzed using content analysis and narrative analysis techniques. According to the results, students appeared to have difficulty solving proportional problems of all types using proportional reasoning. Students are still struggling with distinguishing between proportional and non-proportional situations, as well as direct and inverse proportions. Furthermore, students often encounter difficulties when attempting to solve numerical comparison and qualitative comparison problems. This might be a consequence of students' lack of experience in solving these types of problems. Another tendency is the use of the cross-multiplication algorithm in solving missing-value problems without knowing the purpose of using the algorithm.

Keywords: missing-value; numerical comparison; problem types; proportional reasoning; qualitative comparison

INTRODUCTION

The development and acquisition of reasoning ability is an essential ability for students who want to learn mathematics. NCTM (2000) specifies that students are required to possess the following five skills: problem-solving, reasoning and proof, connections, communication, and representation. The ability of students to manipulate and analyze objects, symbols, representations, diagrams, or statements in order to derive conclusions based on evidence or assumptions defines reasoning ability (NCTM, 2016). With this ability, students are able to give or express reasons for the objects or problems they face.

Students are expected to possess proportional reasoning as one of their reasoning abilities. The concept of proportion can be defined as the statement that two ratios are equivalent, as supported by the works of Ben-Chaim et al. (1998), Kilpatrick et al. (2001), and Langrall & Swafford (2000). Moreover, proportional reasoning refers to the ability of students to use multiplication relationships in understanding and reasoning in situations or problems involving proportions (Im & Jitendra, 2020; Kilpatrick et al., 2001; Langrall & Swafford, 2000; Van Dooren et al., 2018). Therefore, it can be deduced that proportional reasoning ability refers to students' ability to draw conclusions or reason by using

multiplicative relationships when dealing with proportional situations or problems involving two ratios.

In line with the NCTM (2000) content, which specifies that proportional reasoning ability is an essential ability that students must possess, the Ontario Education Ministry (2012) identifies this ability as an essential factor in the development of a student's ability for understanding and applying mathematics. Students must possess this ability in order to understand mathematical concepts including geometry, algebra, and statistics (Beckmann & Izsák, 2015; Kilpatrick et al., 2001; Lobato & Ellis, 2010). Moreover, according to Langrall and Swafford (2000), students who fail to develop this ability often encounter difficulties or obstacles when attempting to understand mathematical concepts at more advanced levels.

Indicators of proportional reasoning ability, which is necessary for students to understand mathematics, include the following (Kilpatrick et al., 2001; Lamon, 2020; Langrall & Swafford, 2000; Septian et al., 2022; Supriyadi et al., 2022; Weiland et al., 2021): 1) students can differentiate between proportional and non-proportional situations, given that not all comparison problems involve proportional situations; 2) students recognize the concepts of covariation in proportional problems; 3) students use multiplicative relationships rather than using additive relationships in solving proportional problems; and 4) students are able to distinguish situations involving direct proportion from inverse proportion. Concerning these indicators, previous studies have indicated that the proportional reasoning ability of students remains weak. This is evidenced by the fact that students keep having difficulty differentiating between proportional and non-proportional situations (Gläser & Riegler, 2015; Karli & Yildiz, 2022), employ additive relationships when solving proportional problems (Ayan & Isiksal-Bostan, 2019; Gündoğdu & Tunç, 2022), and are unable to differentiate between situations involving direct and inverse proportion (Karli & Yildiz, 2022; Mardika & Mahmudi, 2021).

Over the past five years, numerous studies have been done to analyze students' proportional reasoning ability. Prayitno et al. (2019) categorized students based on five levels of proportional reasoning (Prayitno et al., 2018). Putra et al. (2020) described the proportional reasoning ability of students with auditory learning styles in solving proportion material problems. Other studies describe students' proportional reasoning ability by grouping students based on cognitive style (Fadilla & Siswono, 2022; Nur & Sari, 2022; Taufik, 2021; Taufik et al., 2021; Widayanti et al., 2020), gender (Khotimah & Shodikin, 2021), *self-efficacy* (Nurlela et al., 2022), and *adversity quotient* (Khumairoh et al., 2020).

Moreover, there are three types of proportional reasoning problems, namely numerical comparison problems, qualitative comparison problems, and missing-value problems (Johar et al., 2018; Kilpatrick et al., 2001). Kilpatrick et al. (2001) mentioned that the learning process focused on solving missing-value problems and some numerical comparison problems. While the findings of Ayan dan Isiksal-Bostan (2019) indicate that the cross-multiplication algorithm is frequently employed to solve missing-value problems. Yet it should be noted that this algorithm does not automatically indicate that students possess proportional reasoning ability (Lobato & Ellis, 2010; Orrill & Burke, 2013).

Based on those results of previous studies, the researchers believe that it is necessary to examine students' proportional reasoning ability in solving each type of proportional problem; how students solve missing value problems, and whether it is true that students are not familiar with numerical comparison problems or qualitative comparison problems. Yet, there is a lack of study that focuses on this description of students' proportional reasoning ability in solving each type of proportional problem. The previous studies mostly only provide the description of students' proportional reasoning ability in solving one or two of the three types of proportional reasoning problems, which does not give enough information about the comparison of students' ability in solving each type of proportional reasoning problem. This study therefore aims to describe the proportional reasoning ability of students according to each type of proportional problem. Additionally, non-proportional problems were included in this study in order to assess students' ability in the first indicators of proportional reasoning ability.

RESEARCH METHODS

The objective of this qualitative descriptive study is to describe the proportional reasoning ability of students according to each category of proportional problems. The participants of the study were 49 eighth-grade students from one of Bandung City's junior high schools. The instrument used in this study consists of mathematical word problems, which are designed to evaluate proportional reasoning ability. The questions that were included in the instrument are illustrated in Figure 1.

1. Ten years ago, Juan was 3 years old and Jake was two times Juan's age. If Juan is 13 years old this year, then how old is Jake this year?
2. A task can be completed in 24 days by 4 workers. If each worker has the same ability and work speed, then how many workers are needed to complete the same task in 12 days?
3. Every day, Tania and Dina run around a basketball court close to their home.
 - a. Compared to yesterday, today Tania did more laps in a shorter time. How does Tania's running speed today compare to her running speed yesterday?
 - b. If Tania did 3 laps in 4 minutes while Dina did 2 laps in 3 minutes, which of them ran faster?

Figure 1. Questions Given to Students in this Study

The data analysis techniques used in this study are content analysis and narrative analysis. First, students' answers to the questions were analyzed, coded, and categorized using the ATLAS.ti application. This step is categorized as the content analysis technique. The purpose of categorizing student responses is to facilitate the selection process for researchers by identifying those whose responses are most representative of the responses of other students. The technique by which samples are chosen in this manner is referred to as purposive sampling, which involves selecting samples in accordance with specific criteria that are aligned with the research objectives. In addition, nine students were

interviewed in order to gain a deeper understanding of their proportional reasoning ability. The next step is categorized as the narrative analysis technique, which focuses on understanding the explanations shared by the students in the interview session. Following this, the gathered information was thoroughly organized and presented in a descriptive way. The purpose of the way the data is displayed in a descriptive way was to help facilitate the drawing of conclusions as the last step of the data analysis.

RESULT AND DISCUSSION

Students' Proportional Reasoning Ability in Solving a Non-proportional Problem

Question number 1 presents a problem that does not involve a proportional situation. The results show that some students use additive reasoning in their solutions because they recognize that the ratio of Jake and Juan's ages changes from year to year. However, there are also students who rely on the concept of direct proportion because they assume the ratio is constant. The following answer illustrates the work of a student who attempted to solve question number one by applying the concept of direct proportion.

JAWAB
 Dik: 10 thn lalu umur Juan adalah 3 thn dan umur Jake adalah 2x umur Juan
 jika umur Juan tahun ini 13 thn
 dit: berapa umur Jake tahun ini
 Jwb:

umur Juan	umur Jake
3	3 x 2 = 6
13	x

$$\frac{3}{13} = \frac{6}{x} \Rightarrow 3x = 78$$

$$x = \frac{78}{3}$$

$$x = 26$$

Jadi, umur Jake tahun ini adalah 26 thn

Figure 2. Students' Answer in Solving Question Number 1

In Figure 2, students' inability to differentiate between proportional and non-proportional situations, the first indicator of proportional reasoning ability, is demonstrated by their application of the direct proportion concept to solve non-proportional problems. The student S35 used a cross-multiplication algorithm to find the x value from $\frac{3}{13} = \frac{6}{x}$ and got $x = 26$. She claimed that because Jake's age is two times of Juan's age, when Juan is 13 years old then Jake is 26 years old. She failed to understand that the ratio of Jake's age and Juan's age every year is different so the problem cannot be solved by using the direct proportion concept. Several prior studies have similarly discovered that some students continue to employ proportional problem-solving strategies as they deal with non-proportional problems (Gläser & Riegler, 2015; Karli & Yildiz, 2022). In addition, students admitted that they were never presented with this non-proportional problem during learning. The student's statement indicates that they lack the knowledge and experience necessary to differentiate between proportional and non-proportional situations. As a result, they tend to erroneously assume that all presented situations are proportional, making them employ proportional problem-solving strategies to solve non-proportional problems.

Students' Proportional Reasoning Ability in Solving a Missing-value Problem

Question number 2 is a missing-value problem which is also an inverse proportion problem. In this problem, students are presented with a problem related to the relationship between the number of workers, workload, and time needed to complete the work. The results show that some students solve the problem using the concept of direct proportion as shown in Figure 3 below.

JAWAB
 $24 \text{ hari} = 4 \text{ orang pekerja} = \frac{24}{2} = \frac{12}{4} = 2$
 $12 \text{ hari} = 2 \text{ orang pekerja}$

Figure 3. The Use of the Direct Proportion Concept in Solving Question Number 2

The fact that students employ the direct proportion concept to solve problems involving the inverse proportion concept on Figure 3 indicates that they still lack the ability to differentiate between the two concepts. The student determined the workers needed by dividing the number of workers stated in the problem by 2. This is carried out because, in her opinion, a reduction in workers is possible as the time required to complete a task decreases. This case can be related to the students' understanding of the direction of change of two quantities that comprise two proportional ratios. In this context, they make the assumption that the time required to complete the task and the number of workers required both changes in the same direction, while in fact the direction of change of the two quantities is different. Additionally, one student shared in her interview that she had difficulty in identifying proportional problems involving direct proportion and inverse proportion. Similar results have been reported in prior research as well (Karli & Yildiz, 2022; Mardika & Mahmudi, 2021).

Moreover, some students used the concept of inverse proportion to solve problem number two. The answer of one of these students is depicted in Figure 4.

hari	pekerja
24	4
12	x

$$\frac{24}{12} = \frac{x}{4}$$

$$\frac{24 \times 4}{12} = 8$$

Figure 4. Students' Answer to Question Number 2

The answer of one of the students who correctly solved question number two is depicted in Figure 4. In his answer, he found that the task could be completed by 8 workers in 12 days. There were no problems with the method, process, and results that he got. However, the student acknowledged during the interview that he was uncertain as to why the problem ought to be solved in the way that he had written. He also stated that he solved the problem using merely a method he had learned. The solution indicates that the student

solved the problem using the cross-multiplication algorithm. This is consistent with Lobato and Ellis (2010) argument that students' ability in proportional reasoning is not indicated by their use of the cross-multiplication algorithm. Moreover, question number two involving this type of missing-value problem in this study tends to be solved by employing the cross-multiplication algorithm. This finding is consistent with previous studies that have demonstrated that students depend on this algorithm to resolve missing-value problems (Agistnie et al., 2022; Ayan & Isiksal-Bostan, 2019; Mardika & Mahmudi, 2021; Rohmah et al., 2020; Sumartini & Utami, 2023).

Students' Proportional Reasoning Ability in Solving a Qualitative Comparison Problem

Question number 3a is classified as a qualitative comparison problem. In this type of problem, students are given the task of comparing two speeds without being provided with precise numerical values. Students need to understand the multiplicative relationship among time, distance, and speed in order to find a solution to this problem. The following answer illustrates one of the students' answers to question number 3a.

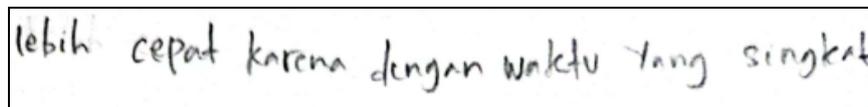


Figure 5. Students' Answer to Question Number 3a

Figure 5 shows the answer of one student who only considers the change of one factor to measure the change in speed. In the given example, the student's conclusion regarding the increase in running speed is based solely on the change in time. This demonstrates that the students are either oblivious to or disregard another factor, namely distance (lap). This is what Karli and Yildiz (2022) refer to as "data neglect."

In addition, some students answered with claims like "the running speed increased because she was diligent" or "the speed increased because she was used to it". Those answers indicate that the students are oblivious to the fact that the presented problem involves mathematical concepts. The Figure 6 below provides an example of an emotional response strategy in question number 3a. Answers like this are referred to as "emotional response strategies" (Karli & Yildiz, 2022).

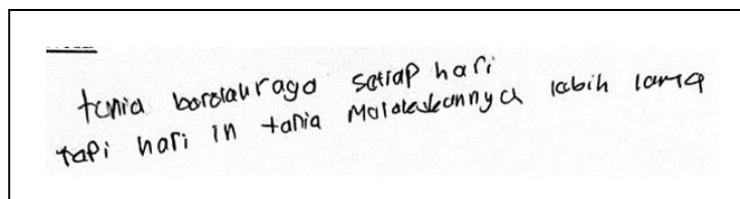


Figure 6. The Example of Emotional Response Strategy to Question Number 3a

Students' Proportional Reasoning Ability in Solving a Numerical Comparison Problem

The problem context of question number 3b is similar to that of question number 3a, which is a proportional problem involving the concept of speed. However, the type of problem presented in problem number 3b is a numerical comparison problem, where

students are tasked with comparing two speeds provided precise numerical values. Some of the students managed to correctly solve this problem, as illustrated in Figure 7.

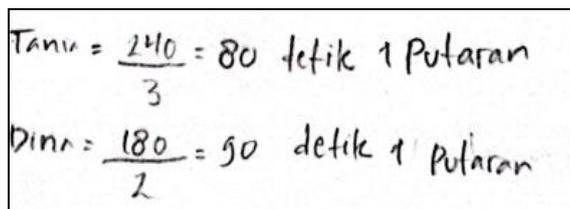
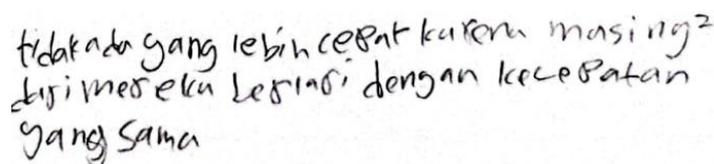

$$\begin{aligned} \text{Tania} &= \frac{240}{3} = 80 \text{ detik } 1 \text{ putaran} \\ \text{Dina} &= \frac{180}{2} = 90 \text{ detik } 1 \text{ putaran} \end{aligned}$$

Figure 7. Students' Answer to Question Number 3b

The student answer shown in Figure 7 is the answer of the same student who only considers the change in time to measure the change in speed in problem number 3a. This shows that students continue to encounter difficulties when attempting to solve problems involving qualitative comparisons. Such difficulties can arise from a lack of familiarity with or experience with solving problems of this type. Furthermore, an issue arises in this numerical comparison problem concerning the students' inability to compare two speeds. This difficulty derives from the students' inadequate ability in the prerequisite topics specifically the comparison of fractions and the conversion of fractions to decimals.

Furthermore, Figure 8 illustrates the answer of a student who claimed that Tania and Dina have identical running speeds. The student stated during the interview that their running speeds are identical due to the fact that both the difference in the number of laps and the time taken by Tania and Dina are the same, with a mere one-point difference. This demonstrates that students solve proportional reasoning problems using additive reasoning. This signifies that students have not yet achieved the third indicator of the proportional reasoning ability. Moreover, prior studies have similarly observed that some students continue to employ additive reasoning to resolve proportional problems (Ayan & Isiksal-Bostan, 2019; Gündogdu & Tunç, 2022).



tidak ada yang lebih cepat karena masing-masing
lari mereka lebih dengan kecepatan
yang sama

Figure 8. The Use of Additive Reasoning in Solving Question Number 3b

CONCLUSION

Based on the results of this study that have been described, it is possible to conclude that students continue to lack ability in proportional reasoning when solving each type of problem. Some students are unable to recognize non-proportional situations when attempting to solve non-proportional problems; consequently, they employ proportional problem-solving strategies to solve the problem. Students frequently employ the cross-multiplication algorithm to solve missing-value problems, which yields the correct solution but fails to explain why the algorithm is used to solve the problem. Additionally, students frequently make mistakes when attempting to identify a proportional problem that involves inverse proportion from direct proportion. The ability of students to solve qualitative comparison problems is also still weak, as demonstrated by certain students who excel at solving numerical comparison problems but struggle with qualitative comparison problems

despite the identical problem context. Lastly, when it comes to solving numerical comparison problems, certain students are still struggling with the prerequisite material, while others rely on additive reasoning to resolve the problem. Such a strategy does not indicate that the students possess proportional reasoning ability.

In addition, the results of this study have some limitations which provide ideas for researchers to conduct studies on students' proportional reasoning ability. The results of this study are only limited to the descriptions of students' proportional reasoning ability in solving proportion material problems, where there are still many other materials related to this ability that can be studied. They also do not provide the description of students' obstacles in learning proportion materials; whether they had difficulties in understanding the material which resulted in their lack of proportional reasoning ability. Furthermore, the description of students' learning obstacles can be used as a basis for teachers in developing learning materials to enhance the students' proportional reasoning ability.

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