MATHEMATICAL SPATIAL REASONING ANALYSIS ON SURFACE OF REVOLUTION AND ELLIPSOIDS USING AUTOGRAPH

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ABSTRACT

The essence of analytic geometry of space course consists in setting up a correspondence between the points of the space and real numbers. With the rapid development of technology, the students need technology supported mathematics learning to solve geometry problems. The aims of this study are to measure students' mathematical spatial reasoning abilities on surface of revolution and ellipsoids using Autograph software. The samples were 43 mathematics education students in second year. This research used quantitative descriptive method. The results showed that 85% of students obtained good category in formula concept of surface of revolution, 80% of students obtained good category on developing object arrangement and parts correlation. 75% of students obtained good enough category on the ellipsoid concept of object rotating and formula on ellipsoid, 65% of students obtained not good enough category on object observing and analyzing from rotating, equations and conclusions. Student attitude showed positive towards surface of revolution and ellipsoids using autograph.

Keywords*: *spatial reasoning, surface of revolution, ellipsoids

INTRODUCTION

Spatial reasoning consists of the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated (Clement and Battista, 1992, p. 1). A person's spatial reasoning can inform ability to investigate and solve problems, especially mathematic problems that are non-routine problems or new problems.

Mathematicians stated that spatial reasoning is an important component of mathematical thinking and mathematical problem solving from pre-school to college level (Cheng and Mix, 2014; Davis et al., 2015; Jones dan Tzekaki, 2016 cited in Kovacevic, 2017). Spatial ability is a concept in spatial reasoning. Linn and Petersen classify spatial abilities into three categories: (1) spatial perception, (2) mental rotation, and (3) spatial visualization (National Academy of Science, 2006).

Based on some research results, in mathematics context, especially the geometry, it is very important to improve students' spatial reasoning. The National Academy of Science (2006) stated that every student should develop spatial reasoning and skills which are very useful in understanding relationships and properties in geometry to solve mathematical problems and problems in everyday life. In mathematics learning process, spatial reasoning is used to learn geometry, because there are elements of visualization, rotation and modeling.

Geometry is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. Geometry is one of the oldest mathematical science. Students should learn geometry by understanding and explaining the physical world by using appropriate problem-solving strategies within (Baki, 2001). Pittalis and Constantinou (2010) state that this type of thinking is "a form of mental activity that enables individuals to create spatial images and to manipulate them in solving various practical and theoretical problems" (p. 191). Student thinking ability in three-dimensional space is basis for spatial reasoning and students in generally have difficulty imagining in relation to analytic geometry of space questions (Schumann, 2003).

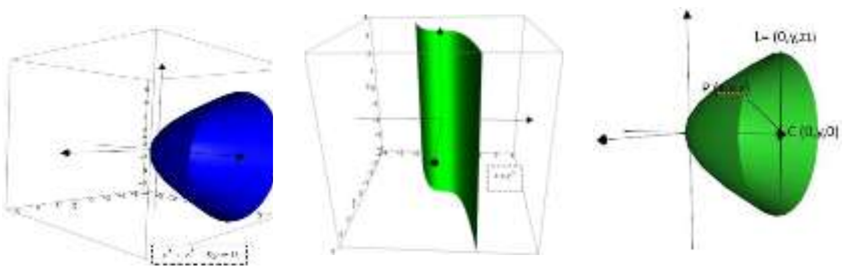
The thinking ability of students in three-dimensional space as a basis for spatial reasoning and research result shows that students have difficulty imagining related to spatial analytic geometry questions generally (Schumann, 2003). Spatial reasoning need visualize three-dimensional objects in the mind and solve mathematical operations. Thus, skills are very important for teaching three-dimensional geometry, because understanding and interpreting a two-dimensional view of three-dimensional objects that in textbooks requires three-dimensional thinking skills. Ertekin (2014) shows that space thinking has a special meaning in spatial reasoning, which includes objects, objects movement of in space (National Research Council, 2006). In other words, visualizing objects movement and manipulating need spatial reasoning development.

Spatial ability and geometry are closely related, therefore spatial reasoning is part of the geometry curriculum. Unal, et al., (2009) stated that students with low spatial abilities had more difficulties in learning geometry. This shows that spatial reasoning is importance in learning geometry. Two dimensional representations for depicting three-dimensional objects causes some problems, as does drawing objects on a whiteboard. Some students have problems drawing and visualizing three-dimensional objects. Kosa, at al (2016) stated that drawing a three-dimensional object in a two-dimensional plane causes vision illusion and differences in perception in seeing object shapes from different angles. Schumann (2003) states that drawing on a blackboard does not offer an opportunity to develop a comprehensive spatial geometry in solving analytic geometry problems. So there needs a device or tools that makes it easier for learners to see three-dimensional objects from different angles by rotating objects, moving objects, and so on.

A tool to ease visualization and help a more flexible representation of objects is to use a computer. Object visualization need computers and graphical calculators and generally students use them to help in the learning process. One of the software used in learning space geometry is Autograph. Autograph is one of the Dynamic Geometry Software (DGS) which is currently growing. This software is widely used by learners to make it easier to understand concepts, prove, and discover concepts. Dynamic Geometry Software (DGS) products aim to develop visualization, discover, and develop mathematical ideas. Therefore, it is important for teachers to understand the use of this DGS in the classroom learning process.

The Autograph has facilities that can encourage user exploration that is good enough to solve problems and find concepts and their relationships. Autograph has a menu in developing visual teaching of three-dimensional geometric objects. Autograph makes it easy for users to construct and manipulate three-dimensional objects on a two-dimensional plane. With Autograph, users can construct objects by entering equations in the input menu, as shown below:

Figure 1. Surface of revolution against Axis



The Autograph facilities will assist students in studying spatial analytic geometry, especially solve problems in rotating surfaces, spheres, ellipsoid, paraboloid and hyperboloid. Visualizing geometric drawing shapes based will assist the student to solve problems.

THEORETICAL FRAMEWORKS

Spatial abilities are defined and evaluated in many ways in the literature. Carol (1993) states that spatial abilities are related to how individuals handle material presented in space or more specifically, the ability to imagine, understand, remember, and change objects or shapes or routes in the real world or through representations of the real world, such as in paper. and pencil or computer tests (Gluck & Fitting, 2003). Megarty & Waller (2005) and Kozhevnikov, et al., (2007) consider spatial abilities as a form of mental activity that allows people to create spatial images and manipulate them in solving various theoretical and practical problems (Pittalis & Christou, 2010)

Linn and Petersen (1985) state that spatial ability is multifaceted because it consists of three types, namely mental rotation, spatial perception, and spatial visualization. This ability was previously thought to be innate, but evidence from experimental research suggests that significant improvement is possible through precise and specific training (Olkun, 2003). The spatial visualization ability refers to the ability to visually manipulate images in space by generating, maintaining, retrieving, and changing whole pattern flexibly and smoothly (Dixon, 1983).

Spatial reasoning can be described as the process of forming ideas through spatial relationships between objects (JMC, 2001). Olkun (2003) defines spatial ability as the ability to manipulate objects and their parts in two and three dimensions. Lohman (2000) shows that spatial abilities consist of 3 separate abilities. These factors contribute equally to the learner's spatial abilities. The three factors are as follows: (a) Spatial Visualization is the ability to understand imaginary movements in three dimensional space or the ability to manipulate objects in the imagination; (b) Spatial orientation is the student's ability to remain unmoved by changing orientations, in which spatial configurations can be represented. (c) Spatial Relationship is defined as the ability to rotate a spatial object as a whole correctly.

Researchers have found that spatial reasoning ability is related to achievement in mathematics, although it shows inconsistencies in some of the cases the researchers' findings have found. Friedman found that spatial and mathematical reasoning had little correlation. He found that verbal ability was a strong predictor of mathematics achievement from spatial reasoning. He also found that spatial math ability correlated stronger among female students than male students. Hanaffin, et al., (2008) found an insignificant relationship between spatial reasoning and achievement and recommended further research in this regard.

Hanaffin, et al. (2008) conducted a study on the effect of students' spatial reasoning and teaching types. This research is supported by the findings of Rhode & Thompson (2007) which state that spatial reasoning in adolescent learning in general can predict cognitive abilities. Pittalis & Christou (2010) investigated the relationship of students' spatial visualization and objects with creative and practical abilities in three-dimensional geometry. The results show that preferences and experiences in spatial visualization are significantly related to students' practical abilities in the three-dimensional cube array. Pittalis & Christou (2010) conducted a study involving students in grades 5 to 9. The results showed that spatial reasoning is a strong predictor of student performance on four types of reasoning in 3D geometry. The findings of this study suggest that an increase in the student's spatial reasoning might result in an increase in his or her 3D geometric reasoning. Battista (1990) points to spatial reasoning as one of the factors influencing success in geometry and geometric problem solving.

With regard to these facts, developing students' spatial reasoning is a must. Smith, et al., (1992) suggests that spatial reasoning can no longer be considered as a narrow aspect, which is only important for certain jobs. Spatial abilities need to be recognized as a fundamental part of intellectual functioning. Because the education system is generally more focused on verbal and numerical abilities, the development of students' abilities in these skills is still lacking attention.

Autograph-math was developed by Douglass Butler. Autograph is a dynamic free and multi-platform math software for all levels of education that combines geometry, algebra, tables, graphs, statistics and calculus in one easy-to-use package. This Autograph software is paid for which can be accessed at http://www.autograph-maths.com/about_autograph/. Some of the advantages of Autograph include: 1) graphs, algebra and tables are connected and very dynamic; 2) Easy to use but manypowerful features, 4) available in many languages. The basic idea of this software is to combine interactive geometry, algebra and calculus in one package that can be used easily for learning mathematics from elementary school to college level.

Some of the uses of the autograph program in learning mathematics are as follows: a. It can produce geometric drawings quickly and accurately compared to using a pencil, ruler, or compass. b. The existence of animation facilities and manipulation movements (dragging) on the Autograph program can provide a clearer visual experience for students in understanding the concept of geometry. c. It can be used as feedback / evaluation to ensure that the painting that has been made is correct. d. Make it easier for teachers / students to investigate or show the properties that apply to a geometric object.

According to Hohenwarter & Fuchs (2004), Autograph is very useful as a medium for learning mathematics with various activities as following. 1. As a media for demonstration and visualization In this case, in traditional learning, the teacher uses the Autograph to demonstrate and visualize certain mathematical concepts. 2. As a construction aid In this case the Autograph is used to visualize the construction of certain mathematical concepts, for example constructing an inner circle or an outer circle of a triangle, or tangents. 3. As a tool for the discovery process In this case, the Autograph is used as a tool for students to find a mathematical concept, for example the position of the points or the characteristics of a parabola.

An interactive way of constructing the use of this software makes it easy to repeat the construction that has been made at any time. Wherever students make additions or deletions of new values on objects, students can see direct changes to these objects that are dependent or bound. starting with simple construction through to function integration. Students can explore mathematics alone or in groups. The teacher acts as a facilitator when help is needed. The results of student experiments with the Autograph should form the basis for class discussion. This gives teachers more time to concentrate on basic ideas and mathematical reasoning (Hohenwarter and Fuchs, 2004).

METHOD

This research is a descriptive study to measure students' spatial reasoning abilities on surface of revolution and ellipsoids using an autograph. Descriptive research is research to describe or describe the existing reality. In principle, descriptive research does not aim at forming hypotheses or developing theories. The samples were 43 people from 4th semester mathematic education students.

The instrument used in this study was an essay made by the researcher. An essay test is a format item that requires a structured explanation in several paragraphs. The characteristics of essay test are containing more than one question that requires a long answer and serious thinking (Kubiszyn and Borich, 2003). The percentage of students' mathematical spatial reasoning ability shown in each indicatoruses the following formula:

Information:

P_i = Percentage of result of indicator's mathematical spatial reasoning ability

$\bar{s}$ = Average student score per indicator

s_i = The ideal score of the intended indicator

Classification of the level of students' mathematical spatial reasoning ability based on test results, using:

$$P_i = \frac{n_i}{N} \times 100\%$$

Information:

P_i = Percentage of students for each mathematical spatial reasoning ability

n_i = The number of students in each mathematical spatial reasoning ability
N = Students total

FINDINGS AND DISCUSSION

Assessment of the results of students' spatial reasoning abilities based on the applicable value provisions in the Faculty of Teacher Training and Education, Suryakancana University, namely:

Table 1. Score dan Interval Value

Raw Score	Interval	Scale
85 -100	3,40 – 4,00	A
70 - 84	2,80 – 3,39	B
60 - 69	2,40 – 2,79	C
50 - 59	0,99 – 2,39	D
0 - 49	0,00 – 0,99	E

Table 2. Description of Test Result DataTest

Data Test	Score		Total
	A	B	
Maximum Score	80	80	80
Minimal Score	54	60	54
Average	68,18	69,45	68,82
Median	69,00	68,00	68,00
Modus	70,00	75,00	65,00
Variance	60,44	31,97	45,55
Deviation Standard	7,77	5,65	6,75

Table 2 shows that although the average score of mathematical spatial reasoning ability achievement in class B is higher than class A, it is still included in the raw score for the category value is B, namely 70-84. This means that students' mathematical spatial reasoning abilities are comparable in class A and B. From the standard deviation, it shows that results achievement of class A are more spread out than class B.

Table 3. Data of Students' Mathematical Spatial Reasoning Ability

Interval	Frequency		Percentage		Overall	
	A	B	A	B	Frequency	Percentage
70 - 84	15	14	68%	67%	29	67%
60 - 69	5	6	23%	29%	11	26%
50 - 59	2	1	9%	5%	3	7%
Total	22	21	100%	100%	43	100%

From Table 3, it shows that the results of students' achievement percentage in mathematical spatial reasoning abilities with grade B are 70 - 84, class A (68%) is higher than class B (67%), C category value are 60 - 69, class A (23%) is lower than class B (29%), and category D scores, namely 50 - 59, class B (9%) is lower than class A (5%).

Overall achievement of students' mathematical spatial reasoning abilities with the B category (70 - 84) were 29 people (67%), the C category were 11 people (26%), the Dcategory (50 - 59) were 11 people (26%), and 3 students (7%). This achievement shows that the average mathematical spatial reasoning ability of students is quite good.

Table 4. Percentage of Students' Mathematical Critical Thinking Ability per Indicator

Indicator	Question Number	Average Score Indicator	Ideal Score Indicator	Percentage Per Indicator
Describes a situation when there is a change or arrangement displacement of two dimensions to three dimensions or vice versa.	1	7.2	4	85%
Understand object arrangement and its parts and their relationship to one another.	2	7.1	4	80%
Explain the concept of three-dimensional rotating objects precisely and accurately.	3	6.4	4	75%
Observe and analyze an object from various circumstances	4	5.43	4	65%
Average				**76%**

The indicator describes a situation while there is a change or arrangement displacement of two dimensions to three-dimensions or vice versa. The one with question number 1 asks the students to answer the equation's rotation about the axis. Students must know the formula for the concept of rotating the surface of an object. The results of the student's ability to achieve an average of 85% are in the good category, meaning that most students can do these questions correctly. This average achievement shows the students are able to identify changes or an arrangement displacement of the second dimension to the third dimension or vice versa.

In question number 2, students have to answer by developing an object arrangement and its parts and their relationship to each other. The results of the student's achievement ability are on average 80% which is in the good category, meaning that most students can do the questions correctly. This average achievement shows that students are able to arrange an object and its parts and their relationship with each other in problem solving.

In question number 3 students had to explain the concept of a dimensional rotating object accurately and accurately on the ellipsoid. Students must apply the formula according to the question. The results of the achievement student's ability an average of 75% are in the good category, meaning that most students can work on these questions correctly. This average achievement shows that students are able to explain the concept of rotating objects on the ellipsoid

In question number 4, students had to look at and analyze an object from various circumstances. Students must determine the formula used, and make conclusions from the results of the rotation equation. The results of the achievement student's ability an average of 65% are in the sufficient category, meaning that most students can do these questions correctly. This average achievement shows that students are able to observe and analyze an object from various circumstances.

Table 5. Student Responses on Rotating Surfaces and Ellipsoids

Scale	Number of Responses toward Statements (%)										Total	
	1	2	3	4	9	13	15	16	18	20		
	+	+	-	+	-	+	-	+	-	+	+	-
Strongly agree	21%	16%	12%	35%	2%	14%	7%	23%	7%	23%	76%	24%
Agree	60%	60%	14%	49%	33%	58%	16%	63%	30%	56%		
Disagree	14%	12%	58%	16%	58%	16%	58%	12%	49%	19%		
Strongly Disagree	5%	12%	16%	0%	7%	12%	19%	2%	14%	2%		

Based on the table above, the average percentage of students' positive attitudes is 76% which indicates that in general the attitude of students towards rotary and ellipsoidal surfaces uses an autograph. Meanwhile, as many as 24% of students gave a negative response to the rotary surface material and ellipsoid using the autograph. Positive statements in this aspect consist of statements number 1, 2, 4, 13, 16, and 20. As many as 81% of students feel happy and excited to follow the rotating surface and ellipsoid material using the Autograph and 19% of students do not like it. In statement number 2, 77% of students liked the material provided and 23% of students did not like the material given. Then in statement number 4, as many as 84% of students agreed that the rotary surface material and ellipsoid using the autograph could be understood by studying it seriously.

In statement number 13, 72% of students stated that the rotary surface material and ellipsoid using the Autograph were quite difficult to understand and 28% of students who did not find it difficult. In statement number 16, as many as 86% of students agreed that the rotary surface material and ellipsoid using the Autograph could be understood by studying it seriously. In statement 20, as many as 79% of students stated that the Rotating Surface is more difficult than the ellipsoid, and 21% stated there was no difference.

Negative statements of this aspect consist of statements number 3, 9, 15, and 18. In statement number 3, a percentage of 74% which indicates that in general students do not agree that the rotary surface and ellipsoid using the Autograph are difficult and tedious subjects and as much as 26% of students agree that mathematics is a difficult and boring subject. In statement number 9, a percentage of 65% which indicates that students do not always want to watch the rotating surface material and ellipsoid using the Autograph in the class to end soon and a percentage of 35% which indicates that they always want to watch the rotary surface material and ellipsoid using the Autograph in class ending soon. In statement number 15, as many as 77% of students disagreed that the rotary surface material and ellipsoid using the autograph were given in a short time, and 23% agreed that they were given in a short time. In question no.18, 63% of students disagreed if the rotating surface material and ellipsoid using the Autograph were too easy to understand, and 37% of students agreed.

Table 6. Student Responses on Mathematical Spatial Reasoning Ability

Scale	Number of Responses toward Statements (%)										Total	
	5	6	7	8	10	11	12	14	17	19		
	-	+	-	+	-	+	-	+	-	-	+	-
Strongly agree	5%	23%	12%	28%	9%	23%	5%	16%	7%	14%	**74%**	**26%**
Agree	14%	58%	23%	58%	28%	49%	30%	60%	12%	19%		
Disagree	58%	12%	56%	14%	53%	16%	53%	16%	74%	53%		
Strongly Disagree	23%	7%	9%	0%	9%	12%	12%	7%	7%	14%		

Based on the table above, that 74% of students agreed that the surface of revolution and ellipsoid exercise questions using the Autograph helped students understand the material provided and 26% of students gave negative responses. Positive statements in this aspect consist of statements number 6, 8, 11 and 14. In statement number 6, 81% of students are like to dothe surface ofrevolution and ellipsoidal exercise questions using the Autograph correctly and 19% of students think normal questions.

In statement number 8, 86% of students indicated that they preferred to do the questions in their own way than seeing their friends 'work and 14% of students preferred to do the questions by looking at their friends' work. In question no.11, 72% of students stated that the questions quite challenged and always want to try to solve the problem in a different way and 28% of students think that no challenge in the questions or always want to try to solve the problem in a different way. In statement number 14, 77% of students stated that the questions quite challenged them, and 23% considered normal questions.

Negative statements in this aspect consist of statements number 5, 7, 10, 12, 17 and 19. In statement number 5, 19% of students do not find it difficult if asked to answer questions with various answers and 81% of students find it difficult if asked to answer questions with various answers. Whereas in statement number 7, 35% of students indicated that they did not take long for questions that were too difficult and 65% of students felt that the questions given were too difficult so that the process took a long time. In statement number 10, 38% of students agree with the statement that students feel bored in spelling out the questions given, and 62% of students do not feel bored. In statement number 12, 35% of students disagree that they do not feel compelled to work on surface of revolution and ellipsoid problems using the Autograph and 65% of students do not feel compelled to work on surface of revolution and ellipsoid problems using the Autograph.

In statement number 17, 19% of students indicated that almost half of the students were able to rewrite the known information on the questions correctly and 81% of students often rewrote the information that was known on the questions. Whereas in statement number 19, 32% of students felt confident in solving problems in their own way and 68% of students felt less confident in solving problems in their own way.

Table 7. Overall Student Attitudes

Measured Aspects	Average Percentage		Interpretation
	Positive Attitude	Negative Attitude	
Students' attitudes towards surface of revolution material and ellipsoids using an autograph	76%	24%	Generally Positive
Students' Attitudes towards Mathematical Spatial Reasoning Ability Questions	74%	26%	Mostly Positive
Total Average	**75%**	**25%**	**Mostly Positive**

Based on the table above, the percentage of positive attitudes of all students is 75% indicates that most students have a positive attitude after getting surface of revolution and ellipsoid material using the Autograph. 76% of students gave a positive response to surface of revolution material and ellipsoid using autographs and 74% of students gave positive responses to problems of mathematical spatial reasoning ability.

CONCLUSION

Based on the research results, the researcher concluded that: 1) the average mathematical reasoning ability of prospective teacher students on the surface of revolution and ellipsoid using autograph is in good category; 2) the average students' mathematical spatial reasoning ability per indicator shows very good category for two indicators, good category for 1 indicator, and enough category for 1 indicator; 3) Positive student attitudes towards surface of revolution and ellipsoid material using an autograph

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